University of Alberta

Three Essays on Bundling

by

Yingtao Shen ©

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Dedication

To my family and friends in China and Canada

ABSTRACT

The first essay compares the profitability of separate auctions of two products to that of a single auction of a bundle consisting of these two items. I add to previous research by examining how (1) the degree of complementarity between the component products, (2) the heterogeneity in bidders' product valuations, and (3) the number of bidders, affect bidders' bidding strategies and the relative profitability of a single auction of the bundle versus separate auctions for the components. I find while one auction of a bundle has inherent inefficiency, in separate auctions of complements bidders bid less aggressively due to exposure risk. The relative profitability of bundling depends on the net effect of these two mechanisms.

The second essay investigates how decision makers make inferences about the value of low-certainty goods based on the value of high-certainty goods. Results of two experiments indicate that bundling a low-value certain item with a high-value uncertain item results in a bundle valuation lower than the value of the uncertain item alone. In addition I find that bundling a high-value certain item with a low-value uncertain item leads to super-additivity, even though the items are not complements. The results demonstrate that departures from additivity are eliminated when ambiguity about the value of the uncertain item is reduced.

The third essay investigates bidders' bidding strategies and the relative profitability of three auction strategies, one auction for the bundle, two simultaneous separate auctions, and two sequential separate auctions under a controlled environment. Both theory and empirical evidence suggest when there is great variation and no asymmetry among product values, the three selling mechanisms are equally profitable. When there is large asymmetry but no variation among product values, bundled auctions are more profitable than two separate auctions when there are two bidders but less profitable when there are ten bidders. Generally, selling products simultaneously or sequentially generates the same revenue.

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Chapter 1

Introduction

Sellers intending to put multiple products up for auction are interested in choosing the optimal (i.e., profit-maximizing) format for auctioning that set of items. An important aspect of this decision is whether to sell a set of products via separate auctions (one for each item) or as a bundle in a single auction. The focus of the first essay is on comparing the profitability of separate auctions of two products to that of a single auction of a bundle consisting of these two items. To that end, I identify conditions under which a single auction of the bundle dominates separate auctions of the component products. I add to previous research by examining how (1) the degree of complementarity between the component products, (2) the heterogeneity in bidders' product valuations, and (3) the number of bidders, affect bidders' bidding strategies and the relative profitability of a single auction of the bundle versus separate auctions for the components. I find while one auction of a bundle has inherent inefficiency, in separate auctions of complements bidders bid less aggressively due to exposure risk. The relative profitability of bundling depends on the net effect of these two mechanisms.

In the second essay of this thesis, I conceptualize, develop and test a multiple-item bundle valuation model through which decision makers are able to make inferences about the value of low-certainty goods based on the value of high-certainty goods. Results of two experiments indicate that bundling a low-value certain item with a high-value uncertain item, which are not substitutes, results in a bundle valuation lower than the value of the uncertain item alone. I refer to this highly unexpected and previously unexplained phenomenon as "hyper-subadditivity." In addition I find that bundling a high-value certain item with a low-value uncertain item leads to super-additivity, even though the items are not complements. In order to manipulate the degree of certainty within items, across experiments, I increase value certainty of the uncertain good in the third experiment. The results demonstrate that departures from additivity are eliminated when ambiguity about the value of the uncertain item is reduced.

An auctioneer of two products has three typical alternatives mechanisms to auction them off: one auction for the bundle, two simultaneous separate auctions, and two sequential separate auctions. The primary objective of the third essay is to investigate bidders' bidding strategies and the relative profitability of these three auction strategies under a controlled environment. I investigate how bidders' bids and seller's revenue are affected by the following three factors proposed in Essay 1. I first derive the equilibrium conditions and next empirically test bidding behavior in the three different auction formats. Both theory and empirical evidence suggest when there is great variation and no asymmetry among product values, the three selling mechanisms are equally profitable. When there is large asymmetry but no variation among product values, bundled auctions are more profitable than two separate auctions when there are two bidders but less profitable when there are ten bidders. Generally, selling products simultaneously or sequentially generates the same revenue.

Chapter 2

Essay 1: The Profitability of Bundling in Auctions

2.1 Introduction

Auction as a pricing mechanism has been used for more than two thousand years (Krishna 2002). In auctions all kinds of product have been sold, including artworks, wines, antiques, FCC spectrum license, and thousands of others. In the past decade, with the advance of technology, auctions have reached millions of bidders through the internet, and hundreds of online auction sites are merely a click away. Internet auctions have gained tremendous popularity and are perhaps the most successful form of e-commerce.

Retailers are also increasingly relying on online auction as an alternative selling mechanism. On the Internet auction website eBay alone, over 724,000 American retailers used auctions as their major channel of distribution, while another 1.5 million individuals supplemented their income by selling on eBay (eBay 2005). This alternative selling mechanism has, however, also created new challenges for these sellers. In particular, auctioneers selling multiple related and unrelated products need to determine the optimum way to sell these items (Cheema et al. 2005). This has raised the important question: under what conditions is the auctioneer better off selling items in separate auctions as opposed to a bundle?

Bundling is a pervasive selling mechanism, and many real world examples can be found, such as seasonal hockey tickets, an automobile with added features, high speed internet combined with cable TV, and numerous other examples.[1] However, while bundling has received considerable attention in a posted-price context, (see Stremersch and Tellis 2002 for a review of bundling in the Marketing and Economics literatures), it has received little attention in a variable-price context.

The limited literature on bundling in auctions suggests that separate auctions tend to be more profitable, especially when the number of bidders is large (e.g., Palfrey 1983, Chakraborty 1999). The intuition can be demonstrated by the following example. Suppose an auctioneer sells two different products, A and B to N bidders in second price sealed bid (Vickrey) auction(s). Also suppose for each bidder, the value of the bundle (V_{bu}) equals the sum of the values of A and B (V_A and V_B). The revenue from an auction of a bundle (R_{bu}) equals the second highest V_{bu}, and the revenue from two separate auctions (R_{se}) equals the sum of the second highest V_A and V_B. Unless the two independent values are perfectly positively correlated, the latter will be greater than the former when there are more than two bidders. Hence, the seller extracts less consumer surplus in the bundled auction, as the winner is the bidder with the highest V_{bu}, but she does not necessarily have the highest valuation for both or even one of the products in the bundle (V_A and V_B). This inefficiency contributes to bundled auction's inferior profitability.

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[1] In this paper we will follow the definition of bundling from Stremersch and Tellis (2002), who define it as "the sale of two or more separate products in one package" (page 56). They referred to "separate products" as products for which separate markets exist. Stremersch and Tellis (2002) also made an important distinction between product bundling and price bundling, with the former meaning "the integration and sale of two or more separate products or services at any price" and the latter meaning "the sale of two or more separate products in a package at a discount, without any integration of the products".

The distribution of bidder's valuations of the two products has the greatest influence on the inefficiency of the bundled auction. The greater the negative correlation between bidders' V_A and V_B, the higher the inefficiency, since the difference between the second highest V_{bu} and the sum of the second-highest V_A and V_B will be greater (see Chakraborty 1999).

However, sellers often have multiple products that are complementary (i.e. $V_A + V_B < V_{bu}$), and the degree of complementarity may be substantial. For example, in FCC auctions of the electromagnetic spectrums, a firm's value for two adjacent licenses (say for the states of California and Nevada) is significantly greater than the sum of the values of two individual licenses from non-adjacent states since there are synergies from operating in two neighboring geographical locations[2]. Examples of complements are also widespread in auctions of collectibles. For multiple stamps which constitute a rare series, buyers' values of the complete series are generally higher than the sum of the separate values of these stamps. People are also generally willing to pay extra for several ancient furniture pieces that match. Complementarity can also be due to savings in transaction costs. For example, in eBay auctions, winners are charged shipping costs. A winner of two eBay auctions run by the same seller may save on shipping costs by having both products sent in the same parcel, especially when the products are small in size and light in weight.

In two separate auctions for complementary products, bidders face a so-called "exposure risk" (Bykowsky et al. 1995; Ausubel et al. 1997; Rothkopf et al. 1998;

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[2] There are increasing returns of two kinds associated with owning multiple licenses: economies of scale in the amount of spectrum covering a particular geographic area; and economic advantages of various types associated with owning licenses that collectively cover large and/or contiguous geographic areas.

Chakraborty 2004). When complementarity is substantial, bidders in separate auctions may bid above their values for one or both of the products in order to increase their chance of winning both auctions and gaining the complementarity. Hence, a bidder runs the risk of winning only one product (and losing the other) at a price above her individual value, or in certain instances she may win both products by paying an amount higher than the value of the bundle. Suppose for a bidder, V_A and V_B are both \$ 20, and V_{bu} is \$ 50. If her bids in a Vickery auction on A and B (b_A and b_B) are respectively \$25 and \$25, and the highest bids (b_A and b_B) from other bidders are \$23 and \$26, she only wins product A by overpaying \$3 dollars. This exposure risk makes strategic bidders bid less aggressively in separate auctions when complementarity is present, and therefore it decreases the relative profitability of separate auctions. Among the more important market forces that affect the exposure risk in separate auctions are the number of bidders, the number of auctions, the magnitude of the complementarity, and the distribution of bidders' values.

So when an auctioneer decides how to auction off two complements, she has to consider the tradeoff between the inefficiency of a bundled auction and the exposure risk of two separate auctions. On the one hand, in the bundled auction since there is only a single bid for the bundle, bidders' weakly dominant bids equal their values of the bundle (in a Vickrey auction). Hence, it is optimum to bid the full amount of the complementarity. This increases the relative profitability of bundling since bidders tend to bid less aggressively in separate auctions due to the potential exposure risk. On the other hand, when V_A and V_B are not perfectly positively correlated, due to the inefficiency of bundled auction mentioned above, the seller does not gain the entire consumer surplus. The relative profitability of bundled versus separate auctions, therefore, depends on the relative magnitude of these two mechanisms.

In this study we investigate several important issues related to auctions of multiple products. The answers to these questions have significant theoretical and managerial implication in marketing. More specifically the following questions are addressed:

For an auctioneer of two complementary products, under what conditions is selling two products in a bundle more profitable than selling them in two simultaneous separate auctions? There are two typical separate auctions for two complements. In two simultaneous auctions, outcomes of two auctions are revealed simultaneously and bidders do not know the outcome of the other auction when they bid in one auction. In two sequential auctions, bidders submit bids after the outcome of the first auction is revealed. Krishna and Rosenthal (1996) show that the two separate auctions are approximately equally profitable. Essay 3 in this thesis also theoretically and empirically proves the equivalency in profitability of these two selling mechanisms. In reality, simultaneous auctions are prevalent. For example, many sellers on eBay set up multiple auctions that end approximately the same time using some auction software. This essay only considers simultaneous auctions in separate auctions.

- How do rational strategic bidders bid in separate and bundled auctions for complementary products? Specifically, for each of these two selling mechanisms, what proportion of the complementarity will be added to their bids?

This study contributes to the literature in the following ways,

1) It is the first study in the auction literature to investigate the impact of the two dimensions of heterogeneity in buyers' independent values, variation and asymmetry, on bidders' strategies and revenues for bundled and separate auctions. This study considers all different combinations of variation and asymmetry in bidders' independent values.

2) This study examines how exposure risk affects different bidders' strategies and seller's revenue in separate auctions. Most papers of auction for multiple products do not consider complementarity (Palfrey 1983, Chakraborty 1999), and Levin (1997) avoids the exposure problem by assuming that bidders' values of the two component products (and the complementarity) are highly positively correlated such that bidder have practically no exposure risk. This study examines how different types of bidders react to exposure risk in a variety of situations.

3) We bridge the gap between the bundling literature and the multi-objects auction literature by providing the boundary condition (defined by the number of bidders, the degree of complementarity, and heterogeneity of individual values) under which bundled auctions are more profitable.

The remainder of this paper is as follows. In section 2, the literature is reviewed. The model and several propositions are discussed in section 3, and we conclude the paper in section 4 with a discussion of the key findings.

2.2 Literature Review

2.2.1 Bundling Literature in Posted Price Context

Since Stigler (1968)'s pioneering paper, bundling has received considerable attention by academics in the field of economics (Adams and Yellen 1976; Schmalensee 1984; McAfee, McMillan, and Whinston 1989; Salinger 1995) and marketing (Guiltinan 1987; Gaeth etc. 1990; Yadav 1994, 1995; Yadav and Monroe 1993; Bakos and Brynjolfsson 1999, 2000; Soman and Gourville 2001; Stremersch and Tellis 2002; Jedidi et al. 2003). This popularity of bundling as a pricing mechanism can be partially attributed to its capability of increasing sellers' profits by extracting more consumer surplus. This is because bundling can reduce the heterogeneity of buyers' reserve prices, by serving as a second-degree price discrimination mechanism.

Adams and Yellen (1976) look at the profitability of bundling for selling two products. They assume that a consumer's reservation price of the bundle is the sum of the reservation prices of the two products. Three selling mechanisms are identified: unbundled sales (individual products are sold separately), pure bundling (all products are sold only as a bundle), and mixed bundling (products are sold individually as well as through a bundle). Using numerical examples, they rank the profitability of the three selling mechanisms and show in general mixed bundling is the most profitable. Schmalensee (1984) has added to the Adams and Yellen's framework by considering continuous distribution of reservation prices. He demonstrates that mixed bundling is a more profitable strategy than either pure bundled or unbundled sales, even when reservation prices are positively correlated.

However, Stremersch and Tellis (2002) argue that in the current bundling literature there exists ambiguity concerning the concept of heterogeneity of reservation prices. They propose that there are two dimensions in the distribution of reservation prices, asymmetry and variation, while previous research has almost exclusively focused on asymmetry. Asymmetry refers to the difference among consumers' reservation prices for the separation products. For two separate products A and B, asymmetry occurs when one segment of buyers has a relatively higher reservation price of A while the other segment has a higher reservation price of B. Variation refers to the difference among consumers' reservation prices for the bundle of products. Asymmetry leads to negative correlation while variation leads to positive correlation. Stremersch and Tellis (2002) show how these two dimensions affect the optimality of bundling in different ways. In both the bundling and multiple unit product literature, heterogeneity of values is given as an environmental factor. There has not been a discussion concerning the causes of variation and asymmetry in the bundling literature. We expect both buyers' knowledge and the information the seller reveals to be important causes. High variation/high asymmetry occurs when people are highly uncertain about the values of both products and the bundle. High variation/low asymmetry may occur when people are highly uncertain about the values of both products but know the values of the two products are similar, for example, two old stamps of equal or similar values in a series. High variation/low asymmetry happens when people have more knowledge about the value of the bundle but are uncertain about values of individual products. For example, an auctioneer may reveal the retail value of a set of furniture to eliminate the uncertainty of the value of the bundle, but buyers may still be uncertain about the values of individual items (ex. chair, table).

Since the bundling literature has identified heterogeneity of consumers' reservation prices (or product valuations) as a key factor determining the profitability of bundling, we will focus on how heterogeneity of bidders' values affects bidders' strategies and seller's revenue. As this study will show, the two dimensions of heterogeneity, asymmetry and variation, have different impacts on the relative profitability of bundling in auctions. It is important to note that the previous research on bundling in auctions has not considered these two dimensions of heterogeneity.

In addition, complementarity[3] of the bundle components can have a major influence on the profitability of bundling (e.g. Lewbel 1985; Matutes and Regibeau 1988, 1992; Telser 1979, Guiltinan 1997, Venkatesh and Kamakura 2003). Venkatesh and Kamakura (2003) find that the optimality of different selling mechanisms (unbundled sales, pure bundling, and mixed bundling) is determined by the degree of complementarity.

It is therefore important to incorporate both aspects of heterogeneity, as well as, complementarity, which most previous papers have ignored.

2.2.2 Auction Literature for Multiple Objects Auctions

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[3] Oxenfeldt (1966) identified eight important sources of complementary of demand: One-Stop Shopping; Impulse buying; Broader Assortment; Related Use; Enhanced Value; Prestige Builder; Image Effects; Quality Supplements relationships. Guiltinan (1997) categorizes complementary into three types: saving purchasing time and effort; enhancing satisfaction with other products; enhancing image of the seller so all products are valued more highly. Venkatesh and Kamakura (2003) show complementarity can affect the optimality of unbundled sales, pure bundling and mixed bundling.

Auctions for a single product have received most attention in the auction literature (see Milgram 2004 for a summary). However, more recently multiple product auctions have become one of hottest areas in auction research and have received academic attention from both marketing researchers (for example, Zeithammer 2005; Cheema et al. 2005) and economists (see Milgram 2004 for a summary). Even through this trend, still very limited attention has been paid to the relative profitability of bundled versus separate attention (see Table 2.1 for an overview of the literature and the components included in these models).

The issue of the relative profitability of bundling versus separate auctions has been discussed for more than twenty years. Contrary to the general finding in the bundling literature in a posted price context, the auction literature suggests that a bundled auction is less profitable than separate auctions for the component products. This is not surprising since bundling in a posted price context in a second degree price discrimination policy and an auction is essentially a first degree price discrimination mechanism.

There are several factors that influence the profitability of bundling: the number of bidders, the degree of complementarity and heterogeneity of bidders' individual values. We will discuss these next.

The existing literature has shown that the number of bidders in auctions is an important determiner of the relative profitability of bundling in auctions. Palfrey (1983) was the first to compare the profitability of one bundled Vickrey auction versus two simultaneous separate Vickrey auctions for the component products. He considered the case when bidders' values of the component products are independently distributed. He showed that when there are more than two bidders, the bundled auction is less profitable than separate auctions. However, the difference between revenues of these two selling mechanisms vanishes when the number of bidders goes to infinity. Chakraborty (1999) adds to this conclusion by showing that for two non-complementarity products, whose values are independently distributed, there is always a threshold for the number of bidders above which separate auctions will always be more profitable. So in general these papers have concluded that without complementarity separate simultaneous auctions are more profitable than bundled auctions for more than two bidders.

However, which auction format will be more profitable for complementary products? Levin (1997) and Krishna and Rosenthal (1996) considered complementarity of multiple product auctions; however, neither of these papers compared the revenue of bundled versus separate auctions. Levin (1997) characterizes the optimal auction mechanism for selling two complements. He concludes that generally bundling is not optimal. However, he makes a very strict assumption that bidders' individual values of the two component products and the value of complementarity are all driven by the same signal, such that all bidders will honestly reveal their true values.[4] Krishna and Rosenthal (1996) assume there are two kinds of bidders, global bidders and local bidders. Local bidders only want to bid on one of the products and, hence, receive no complementarity. Each global bidder has equal values for the two component products and the value of the bundle exceeds the sum of the individual values for the individual products. They show as the number of global bidders increases, global bidders bid less aggressively in separate auctions.

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[4] The signal ensures that the optimal auction is "incentive compatible", and that "voluntary participation" and "individual rationality" are satisfied. However, Levin (1997) admits that the optimal auction he comes up with "requires too much information to be feasibly implemented except in special cases" (page 190)

A close examination of the previous papers indicates that the conclusion of low profitability of bundled auctions is largely due to either the assumption of non-complementarity (Palfrey 1983, Chakraborty 1999), or specific distributions for the individual values of the multiple products (Levin 1997), and/or insufficient consideration of bidders' strategies when complementarity is present (Levin 1997).

Based on the above discussion, we will next propose a model that will incorporate the following components: 1) The two dimensions of heterogeneity in buyers' independent values, variation and asymmetry 2) Complementarity for the component products 3) The number of bidders

Table 2.1 Overview of Papers on Bundling in Auctions

Models	Complements	# of bidders	Heterogeneity of bidders values	Bidder's strategies	# of products in a bundle
Palfrey (1983)	No	≥2	Independently distributed	No	≥2
Chakraborty (1999)	No	≥2	Independently distributed	No	=2
Chakraborty (2006)	No	≥2	Independently distributed	Limited	=2
Levin (1997)	Yes	≥2	Driven by same factor	Limited	=2
Current model	Yes	≥2	Both variation and symmetry	Yes	=2

This paper does consider combinatorial auction as an alternative selling mechanism. Porter et al (2003) argue that there are three major reasons for rarity of combinatorial auctions in practice. (1) Computational uncertainty: The selection of the winning bids and what it would cost for competition to displace them typically requires the solution of integer-programming problems. There is no guarantee that the solution for such a problem can be found in a ''reasonable'' amount of time when the number of bidders and items becomes larger. (2) Bidding complexity: Combinatorial auctions would be burdensome and difficult for participants and the auctioneer. This is because there are inconceivably many packages on which a bidder might want to place bids, and selecting any subset may be strategically awkward and provide the auctioneer with incomplete information. Also there is a computational problem for the bidder to determine how much to bid to be successful. (3) Threshold problem: Krishna and Rosenthal (1996) show that this threshold problem makes combinatorial auctions less profitable than sequential and simultaneous auctions. Suppose each of two small bidders is bidding on a separate item, but a third bidder is bidding on a package that contains both items. Then the two small bidders must implicitly coordinate through their bidding to ascertain what price each will pay in order for the sum of both bids to exceed the package bid.

2.3 The Model

2.3.1 The Auctions

A revenue-maximizing monopolist has one unit of two products, A and B, which can be either identical or non-identical, to auction off to N bidders (N≥2) in either one auction as a bundle or two simultaneous separate second price sealed bid (Vickrey) auctions[5]. In the auction of the bundle, each bidder submits one bid (b_u) for the bundle. The bidder with the highest b_u wins, and the price she pays equals the second highest bid.

In the two simultaneous separate auctions, each bidder submits two bids, (b_A, b_B) respectively for product A and B. The outcomes of these two auctions are decided simultaneously. In each of the two separate auctions, the bidder with the highest bid wins, and the price she pays equals the second highest bid.[6] To simplify our analyses, resale of the products is not considered in this study.

The following assumptions are made in this study:

- Both seller and bidders are risk-neutral.
- The number of bidders (N) is the same in the two separate auctions and the bundled auction. The number of bidders is common knowledge to all bidders and the seller.
- We assume that all bidders have a positive value for both products A and B; hence, they are all global bidders, rather than local bidders who only have a

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[5] A major reason for using a Vickrey auction is that without complementarity, bidders' weakly dominant strategy is to bid value for the products, regardless of their risk attitude or the number of competing bidders. Most previous research has also used Vickrey auctions (e.g. Krishna and Rosenthal 1996; Palfrey 1983; and Chakraborty 1999).

[6] The following rules are employed to handle ties. In the bundled auction, when there is a tie, the high bidders (say *m* bidders) have an equal chance (i.e. chance of *1/m*) to win the bundle. In the two separate auctions, suppose there are *m* highest bidders on A, *n* highest bidders on B, and *j* highest bidders on both A and B. Please note that the *j* bidders are the overlap of the *m* and *n* bidders. When *j*=0, each of *m* bidders has a chance of *1/m* to win A and each of *n* bidders has a chance of *1/n* to win B; when *j*>0, each of *j* bidders has a chance of *1/j* to win both A and B.

positive valuation for one of the products (see, e.g., Krishna and Rosenthal 1996)[7].

- Complementarity (C) of product A and B is the same for all bidders, regardless of their individual values of A and B (V_A and V_B)[8]. C is common knowledge to all bidders and the seller. C is standardized in the same units as V_A and V_B, where $0 < C < \infty$.

- A bidder's value of the bundle of A and B (V_{bu}) equals the sum of her individual values of A and B (V_A and V_B) and the complementarity.

- Each bidder's V_A and V_B are privately known and are realizations of the same distribution that is common knowledge to all bidders and the seller.

The distribution of values of V_A and V_B is characterized by four different combinations of V_A and V_B, which denote four bidder segments or different types of bidders. Each bidder has a 25% chance of being chosen by nature to be of one of the four types (T1 to T4). Figure 2.1 shows the distribution of the individual values V_A and V_B for each of the type of bidders. The X-axis shows the valuation of V_A while the Y-axis the valuation of V_B. The distribution is standardized such that the maximum values of V_A and V_B are always 1. Table 2.2 shows the amounts of the four types' V_A and V_B when they are either positively correlated (the second column) or negatively correlated (the third column). We define a variable L to describe the heterogeneity between V_A and V_B. L varies from -0.5 (when V_A and V_B are perfectly negatively correlated) to 0.5 (when V_A and V_B are perfectly positively correlated). When L > 0, L determines the values of T2 and T4 bidders, which are (L,1-L) and (1-L, L) respectively. When L < 0, L determines the values of T1 and T3 bidders, which are (1+L, 1+L) and (-L, -L) respectively.

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[7] Note that the distribution of V_A/V_B is standardized to the interval [0, 1], and therefore, it is possible, after standardization, that bidders with zero V_A or V_B have a positive value for C.

[8] Krishna and Rosenthal (1996) also make this assumption while Levin (1997) and Venkatesh and Kamakura (2003) assume that C increases as V_A and V_B increase. Applicability of each of the two different assumptions is case sensitive. In many instances, while people may have different valuations V_A and V_B, they have similar values for the complementarity. For example, a winner of two items sold by the same seller can save similar amounts on shipping, when items are shipped together. In this case the complementarity is the same regardless of V_A and V_B. In some cases, complementarity is expected to be greater when V_A and V_B are greater (e.g. the complementarity of two valuable stamps in a series).

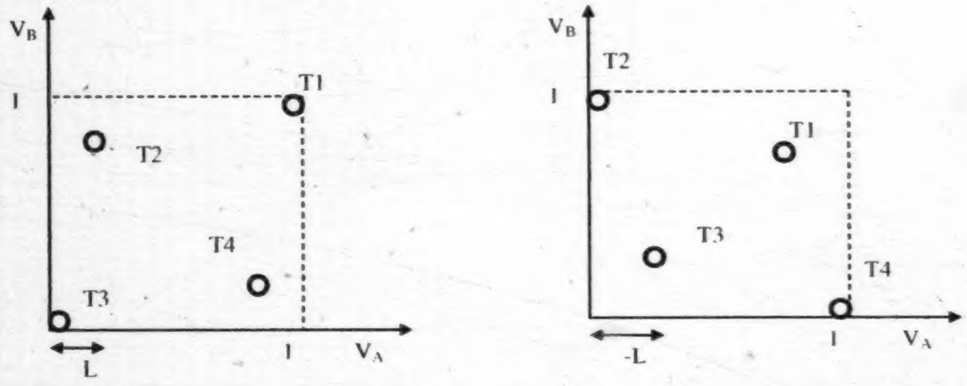
The two dimensions of heterogeneity in buyers' individual values are variation which represents the difference between V_A and V_B, and asymmetry which represents the difference between (V_A+V_B). A major advantage of this discrete joint distribution of the two individual values is that the two dimensions of heterogeneity of V_A and V_B can be easily manipulated. When two values are positively correlated (when L > 0) by increasing L, we can decrease asymmetry without changing variation by moving the values of T2 and T4 closer together; when two values are negatively correlated, (when L < 0), by decreasing L, we can decrease variation without changing asymmetry by moving the values of T1 and T3 closer together (see Figure 2.1). Therefore, this distribution allows us to examine the impact of each of the two dimensions of heterogeneity on bidders' strategies and seller's revenues.

Furthermore, very importantly, based on this discrete joint distribution we can replicate the major conclusions from Palfrey (1983) and Chakraborty (1999) that are essentially special cases of the current model.(i.e. L=0).

Table 2.2 Joint Distribution of Individual Values of Two Products

Type	(V_A, V_B), when $L \geq 0$	(V_A, V_B), when $L<0$
T1	(1, 1)	(1+L, 1+L)
T2	(L, 1-L)	(0, 1)
T3	(0, 0)	(-L, -L)
T4	(1-L, L)	(1, 0)

Figure 2.1 Distributions of Two Individual Values V_A and V_B

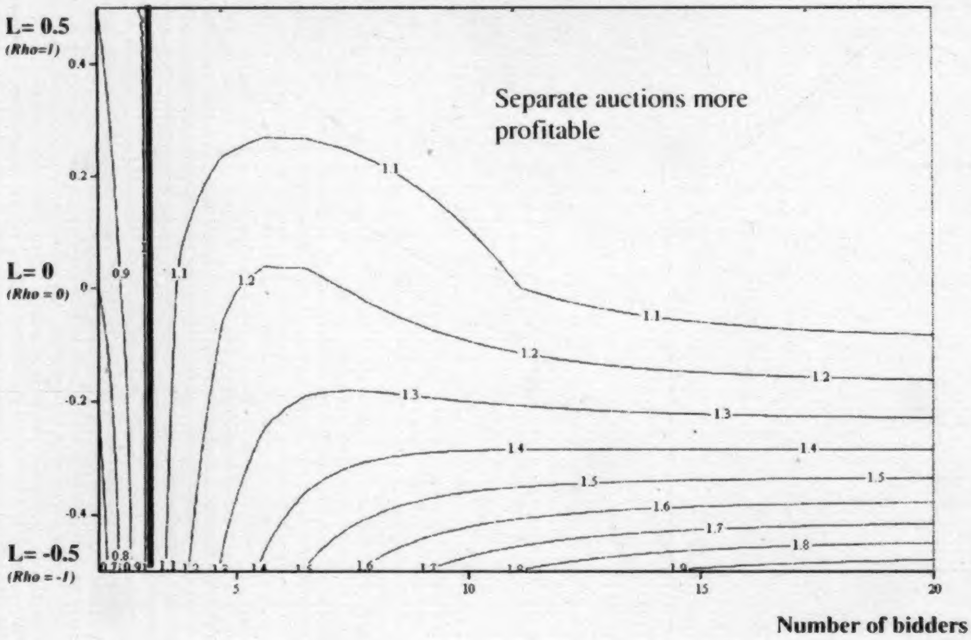


1(A): High Variation, Low Asymmetry **1(B): Low Variation, High Asymmetry**

We first consider auctions without complementarity (e.g. Palfrey 1983, Chakraborty 1999). When complementarity is not present, bidders' optimal bids simply equal their values, V_A and V_B, in the two separate auctions and their value (V_A+V_B) in the bundled auction. The revenues of both separate and bundled auctions are therefore decided by the realizations of the above distribution of individual values and the number of bidders. The expected revenues are the weighted sum of revenues of all possible outcomes. Appendix 2.1 shows how the revenues are calculated in both separate and bundled auctions. Figure 2.2 is a contour plot showing the boundary conditions for profitability of the bundled auctions versus separate auctions. The contours in Figure 2.2 represent the ratios of the expected revenue of two separate auctions to the expected revenue of one auction of the bundle, for different combinations of the number of bidders (N) and different degrees of heterogeneity (L). Consistent with the results of Chakraborty (1999) we find that there is a threshold number of bidders (around 3 in most cases) above which separate auctions are always more profitable. Also, consistent with Palfrey (1983), who assumes that the correlation between V_A and V_B is zero (L =0), we find that when L=0 bundling is more profitable for two bidders but becomes less profitable for three or more bidders. However, the difference between the revenues of the two selling mechanisms vanishes as the number of bidders increases. Therefore, the calculation of revenues using a more parsimonious four-type discrete distribution of individual values, instead of a continuous distribution used by both Palfrey (1983) and Chakraborty (1999), confirms the main finding of these two studies.

In the remainder of this paper our model compares the profitability of one bundled auction versus separate auctions. In both bundled and separate auctions bidders' bids are calculated according to a Bayesian Nash Equilibrium (BNE), while varying the number of bidders (N), complementarity (C) and the distribution of V_A and V_B. The seller, who knows each type of bidders' type contingent bids in the BNE, will calculate and compare the expected revenues of the two selling mechanisms. We determine the boundary conditions under which bundled auctions are more profitable. To do so, we calculate the ratio of the revenue of a bundled auction to the revenue of separate auctions for all given combinations of N, C and distribution of V_A and V_B.

Figure 2.2 Ratios of the Revenue of Separate Auction to Revenue of an Auction of the Bundle without Complementarity



2.3.2. The Equilibrium Bids

2.3.2.1 One Bundled Auction

For an auction of the bundle of A and B, the following result can by derived from our assumptions.

Propostion 1*: in a Vickrey auction of the bundle, a bidder i's weakly dominant bid equals her value for the bundle* V_{bu}*, i.e.,* $(V_{Ai}+V_{Bi}+C)$*, where* $i=1, 2,\ldots, N$*, N is the number of bidders,* V_{Ai}, V_{Bi} *are bidder i's individual values of A and B, and C is the value of complementarity.*

Proof: The bundled auction is actually an auction for just one product, i.e., the bundle. Therefore, it is optimum for a bidder to bid her value of the bundle (see Krishna 2002).

2.3.2.2 Strategy Squares in Two Simultaneous Separate Auctions

Let us first look at bidders' weakly dominant strategy set for two simultaneous separate auctions. The following result can be derived from our assumptions.

Proposition 2*: in two simultaneous separate auctions, bidder i's (i=1, 2,..., N) weakly dominant bids (*b_{Ai}*,* b_{Bi} *) satisfy*

$$V_{Ai} \leq b_{Ai} \leq V_{Ai} + C,$$

$$V_{Bi} \leq b_{Bi} \leq V_{Bi} + C,$$

where V_{Ai} *,*V_{Bi} *are bidder i's individual values of A and B, C is the value of complementarity, and* b_{Ai} *,*b_{Bi} *are the two bids of bidder i in the two separate auctions*

Proof:

To prove that in two simultaneous separate auctions, bidder *i*'s (i=1, 2,..., N) weakly dominant bids satisfy: $V_{Ai} \leq b_{Ai} \leq V_{Ai} + C$ and $V_{Bi} \leq b_{Bi} \leq V_{Bi} + C$, we first need to show that for each strategy that does not satisfy this condition, there is one strategy that satisfies the condition (inside the square) which is at least as profitable. Graphically, bidder *i*'s weakly dominant strategies lie in a "strategy square," with origin (V_{Ai} , V_{Bi}) and width C (e.g. see Figure 2.3). Therefore we need to prove for each strategy outside the square, that there is a strategy inside the square (including the border) that is at least as profitable.

For any strategy, say a strategy (b_{A1}, b_{B1}), which is located outside the strategy square, there are five possibilities:

(1) $V_{Ai} + C < b_{A1}$, $V_{Bi} + C < b_{B1}$;

(2) $V_{Bi} + C \geq b_{B1} \geq V_{Bi}$, $b_{A1} \geq V_{Ai} + C$ or $V_{Ai} + C \geq b_{A1} \geq V_{Ai}$, $b_{B1} \geq V_{Bi} + C$;

(3) $b_{A1} \geq V_{Ai} + C$, $V_{Bi} > b_{B1}$ or $b_{B1} \geq V_{Bi} + C$, $V_{Ai} > b_{A1}$;

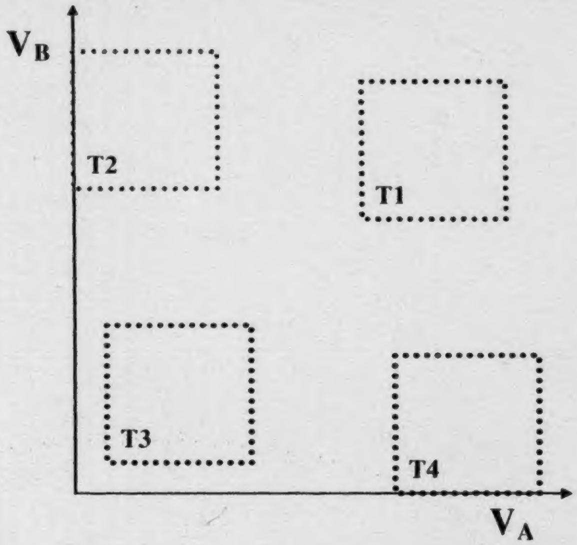
(4) $V_{Bi} > b_{B1}$, $V_{Ai} + C \geq b_{A1} \geq V_{Ai}$ or $V_{Ai} > b_{A1}$, $V_{Bi} + C \geq b_{B1} \geq V_{Bi}$;

(5) $V_{Ai} + C > b_{A1}$, $V_{Bi} + C > b_{B1}$;

For each of these five areas we can show that there is a strategy inside the square that is at least as profitable as this strategy outside the square (for details of proof see Appendix 1).

As shown by proposition 2, the strategy of each type of bidder can be denoted by a square, which we call their strategic square (see Figure 2.3). At the origin of the strategic square are the product valuations V_A and V_B of a bidder, and the size of the square is defined by the magnitude of the complementarity. Therefore, given that there are four types of bidders, there are always four strategy squares with the same width but at different locations, depending on the distributions of individual values.

Figure 2.3 Four Types of Bidders and Their Strategy Squares



Proposition 2 shows that in a BNE, each type of bidders' type contingent bids should be in their "strategic squares." So what we need to know next is where the type contingent bids are located in their strategy squares for the two separate auctions. Or in other words, how much of the complementarity will bidders add to each of the individual products. To determine this we will next calculate bidders' BNE type contingent strategies.

2.3.2.3 Bidders' Type Contingent Bids

In two simultaneous auctions, each strategic bidder *i* (*i* = 1,…,N) will choose her b_A and b_B in her strategy square, such that she maximizes her expected surplus, which is specified as follows:

$$\pi = P_{AB} \cdot (V_A + V_B + C - b_{Ah} - b_{Bh}) + P_A \cdot (V_A - b_{Ah}) + P_B \cdot (V_B - b_{Bh}) \quad (1)$$

Where P_{AB} is the probability of winning both A and B

P_A is the probability of winning A and losing B

P_B is the probability of winning B and losing A

b_{Ah} is the 2nd highest b_A (submitted by other bidders)

b_{Bh} is the 2nd highest b_B (submitted by other bidders)

Essentially, for bidder *i* the maximization task is a tradeoff between winning both A and B, and therefore receiving the complementarity, with positive profit (the first item in equation (1)), and winning only one product with a potential loss (the second and third items in equation (1)). To determine these bids we will calculate the BNE. In the BNE, each bidder's (of any type) profit is maximized given the other bidders' strategies, such that no bidder has incentive to depart from this BNE. The equilibrium bidding strategy is obtained by solving the first-order conditions of the bidder's maximization problem specified in equation (1).

However, one the most significant difficulties in studying multi-unit auctions is the inability to solve the equilibrium bidding strategies as closed form expressions (Chakraborty 2004). Consistent with Chakraborty (2004), we do not find a single universal closed form solution for the equilibrium bidding strategies for all combination of C, N and L. Instead, bidders' type contingent strategies may not change smoothly as C, N and L vary.

The non-existence of a universal BNE for our model is largely due to the discontinuity of T1 and T3 bidders' strategies. As the rest of this section will show, a slight change in the combination of the locations (i.e., heterogeneity of individual values decided by L) and the size (decided by complementarity C) of the four strategy squares could result in great change in T1 and T3 bidders' strategies.

For example, when L < 0, T1 bidders have a higher V_A than T2 bidders, and a higher V_B than T4 bidders, but T1's value of the bundle is less than the sum of T2's V_A and T4's V_B. Therefore in a BNE, T1 bidders have to decide whether they need to outbid both T2 and T4 bidders in both auctions. For some combinations of L and C (e.g. scenario 1 in Table 2.3), T1 bidders can outbid all T2, T3, and T4 bidders and always have positive profit. However, when L decreases a little, to the extent that T1 bidders can not outbid both T2 and T4 in both auctions (e.g. scenario 2 in Table 2.3), T1 bidder can only either outbid T2 or T4 bidders in one of the two auctions. Therefore T1 bidders' strategy may not change "smoothly" as L varies. So we have to examine the BNE for all possible cases.

Based on bidders' strategies in a BNE, we can put all possible combinations of L and C into eight exhaustive and mutually exclusive scenarios six when L/Rho ≤0 and two when L/Rho > 0 (see Table 2.3 and see Appendix 3 for a graphical demonstrations of the eight scenarios). For each scenario, the four types of bidders' strategies can be expressed by the same mathematical format as L, C and N vary.

Table 2.3 All Possible Combinations of L and C

	Scenario	L	C
$L \le 0$	1	$0 \le -L \le 0.5C$	$0 \le C \le 0.5$
	2	$0.5C \le -L \le C$	$0 \le C \le 0.5$
	3	$C \le -L \le 0.5$	$0 \le C \le 0.5$;
		$0 \le -L \le 0.5C$	$0.5 \le C \le 2/3$;
	4	$0.5C \le -L \le 0.5$	$0.5 \le C \le 2/3$
		$0 \le -L \le 0.5C$	$2/3 \le C \le 1$
	5	$0.5C \le -L \le 0.5$	$2/3 \le C \le 1$
	6	$0 \le -L \le 0.5$	$1 \le C \le +\infty$
$L > 0$	7	$0 \le L \le 0.5$	$0 \le C \le 0.5-L$
	8	$0 \le L \le 0.5$	$(0.5-L) \le C \le +\infty$

The analysis of the BNEs for the scenarios above, are shown in Appendix 4. Based on the analysis we obtain the following result for the four types of bidders' type-contingent strategies.

***Proposition 3**: the four types of bidders' type contingent bids in a Bayesian Nash Equilibrium for the eight possible scenarios are as specified in Table 2.4.*

The eight scenarios can be put into three categories according to how bidders answer the question, "How much of C should I add to my bids for each of the products?", for different values of L, N, and C. In other words we categorize bidders based on their

BNE type contingent strategies (those that have the same mathematical expression as indicated in Table 2.4).

In the first category are Scenarios 1, 3, 5, and 7, let us first look at where T2 bidders will bid in their strategy square. Figure 2.5 provides an analysis of profitability of the bidding strategy for a T2 bidder. Figure 2.5(A) shows that for a given strategy (b_{AT2}, b_{BT2}), represented by the arrow, there are six possible outcomes, depending on her opponents' highest bids on A and B (b_{Ah} and b_{Bh}). The profits of the T2 bidder will, of course, depend on the highest bids b_{Ah} and b_{Bh} from her opponents, which vary for the different zones as follows:

- (Zone 1) When $b_{Ah} < b_{AT2}$ and $b_{Bh} \leq b_{BT2}$, or $b_{Ah} \leq b_{AT2}$ and $b_{Bh} < b_{BT2}$, she wins both A and B, and gains a profit of $(V_{AT2}+V_{BT2}+C-b_{Ah}-b_{Bh})$;
- (Zone 2) When $b_{Ah} > b_{AT2}$ and $0 < b_{Bh} \leq V_{BT2}$, she wins A and gains a profit of $(V_{BT2}-b_{Bh})$;
- (Zone 3) When $b_{Ah} > b_{AT2}$ and $V_{BT2} < b_{Bh} < b_{BT2}$, she wins B and receives a loss of $(V_{BT2}-b_{Bh})$;
- (Zone 4) When $b_{Bh} > b_{BT2}$ and $V_{AT2} < b_{Ah} \leq b_{AT2}$, she wins A and receives a loss of $(V_{AT2}-b_{Ah})$;
- (Zone 5) When $b_{Ah} \geq b_{AT2}$ and $b_{Bh} > b_{BT2}$, or $b_{Ah} > b_{AT2}$ and $b_{Bh} \geq b_{BT2}$, she loses both A and B and has zero profits;

- (Zone 6) When $b_{Ah} = b_{AT2}$ and $b_{Bh} = b_{BT2}$, she always has zero profit no matter whether she wins or loses A and B;

Please note the profit in (zone 1) is greater than that in (zone 2) since the value of $V_{AT2}+C-b_{Ah}>0$, is greater in (zone 1) than in (zone 2). Obviously, for a T2 bidder an "ideal" strategy will be one that maximizes her chance in (zone 1) and (zone 2) and minimizes her chance in (zone 3) and (zone 4).

Figures 2.4(A) and (B) compare a strategy for a T2 bidder of spreading C over the bids of A and B (as indicated by the direction of the arrow in Figure 2.4(A)) versus a strategy of adding all C to the bid of the low value product A (see arrow in Figure 2.4(B)). Comparing the different zones in the two figures, we can see that the strategy in Figure 2.5(B) is preferred, as it removes the exposure risk for B (eliminates zone 3) and increases the size of the most profitable zone (zone 1).

First let us look at the T2 bidders' b_B. Comparing Figures 2.4(A) and (B), we can see bidding V_{BT2} ($b_{BT2'}$) weakly dominates bidding higher than V_{BT2} (b_{BT2}). By bidding above V_{BT2} the T2 bidder 'moves into' zone 3 and increases the exposure risk for B. This is because T1 bidders' strategy square, which lays to the right of T2 bidders' square, cover part of zone 3, i.e. when T1 bidders are present, there is a positive chance for a T2 bidder to win B alone at a loss (while a T1 bidder wins A). This chance can be reduced by reducing b_B. In Figure 2.4(B) when $b_{BT2'} = V_{BT2}$, zone (3) actually disappears, and T2 bidders do not need to worry about the possibility of winning B alone by overpaying. In addition, changing b_B to V_{BT2} doesn't affect T3 bidder's profit when there are T3 and T4 bidders.

Table 2.4 Bayesian Nash Equilibrium Bids for All Combinations of L and C

	Type 1		Type 2		Type 3		Type 4	
Scenario	b_A	b_B	b_A	b_B	b_A	b_B	b_A	b_B
1	l+L+0.5C	l+L+0.5C	C	l	-L+0.5C	-L+0.5C	l	C
2	l	l or l+2L+C*	C	l	-2L or C*	C	l	C
3	l+L+0.5C	l+L+0.5C	C	l	-L+0.5C	-L+0.5C	l	C
4	l	l or l+2L+C*	C	l	-2L or C*	C	l	C
5	l+L+0.5C	l+L+0.5C	C	l	-L+0.5C	-L+0.5C	l	C
6	l+L+0.5C	l+L+0.5C	0.5+0.5C	0.5+0.5C	-L+0.5C	-L+0.5C	0.5+0.5C	0.5+0.5C
7	l+ 0.5C	l+ 0.5C	L+C	l-L	0.5C	0.5C	l-L	L+ C
8	l+L+0.5C	l+L+0.5C	0.5+0.5C	0.5+0.5C	-L+0.5C	-L+0.5C	0.5+0.5C	0.5+0.5C

**Note: the choice of the two alternative type contingent strategies depends on C, L and N, for details see the Appendix 4.*

Proof: see Appendix 2.4.

For T2 bidders, bidding $b_{AT2'}$ $(V_{AT2}+C)$ on A is better than bidding less than that. Because the T3 bidders' strategy square covers both zone 1 and zone 2, T2 bidders can outbid T3 bidders on both A and B or only B with positive profit. Please recall winning both A and B is more profitable than winning only B. So by increasing b_A, T2 bidder increases the chance of outbidding T3 bidder on both A and B with a profit (i.e. let a greater area of the T3 bidders' square fall into zone 1). Therefore T2 bidders' best strategy is to bid at the right corner of the bottom of T2 bidders' strategy square ($b_{AT2'}$, $b_{BT2'}$). Similarly, T4 bidders will put the entire C on b_A and just bid her V_B on B.

Therefore due to the exposure risk (winning only one product and pay a price higher than the value of the product), T2 and T4 bidders only put C on the low value product. These strategies of T2/T4 bidders, as opposed to putting more C on the higher value product, decrease the profitability of two separate auctions. This is so, since higher value bidders have a greater influence on the revenue of a second price auctions than low value bidders. For example, when T2 bidders, who have a relatively high V_B, put the entire complementarity on b_A, the positive effect of the complementarity on the revenue of separate auctions is smaller than when T2 bidders put more complementarity on their bids on B.

"Knowing" T2/T4 bidders' strategies and having the highest V_A and V_B, T1 bidders do not need to worry about exposure risk. They will bid ($V_{AT1}+0.5C$, $V_{BT1}+0.5C$) to safely outbid T2, T4 and T3 bidders[9]. There is no incentive for a T1 bidder to bid higher, because if she is the only T1 bidder, bidding higher does not increase profit since her profit is not affected by her bids; when there are *m* T1 bidders, each bidder has 1/*m* of chance to win both A and B and each T1 bidder's expected profit is zero, hence bidding higher does not increase profits.

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[9] Actually T1 bidders' BNE strategy can be any strategy ($V_{AT1}+0.5C+\theta$, $V_{BT1}+0.5C-\theta$), where θ is any number, given that $b_{AT1}>b_{AT4}$ and $b_{BT1}>b_{BT2}$. Since for all these strategies $b_{AT1}+b_{BT1}=V_{AT1}+V_{BT1}+C$, given other types' BNE strategies, they affect T1 bidders revenue of the two separate auctions in the same way. Therefore, we assume T1 bidders add 0.5C to both bids for A and B.

T3 bidders, when faced with competition from other types of bidders, can not win either one or both A and B with positive profit. T3 bidders' bids are the lowest of all types, and they have no incentive to increase their bids. Only when all her (N-1) opponents are of T3, a T3 bidder may have 1/N chance to win both A and B with a profit of zero, and overbidding can make her win A and B but will not increase the profit.

Figure 2.5 summarizes the BNE strategies for the scenarios of category 1, for the different type of bidders. The arrows in these squares represent the four types of bidders' type contingent strategies. Due to the exposure risk, T1 and T3 bidders will just add half of the complementarity to each of their two bids, while T2 and T4 bidders add the entire complementarity to their bid for the low value product. In these four scenarios both asymmetry and variation of individual values are high.

In the second category are scenarios 2 and 4. In these two scenarios, asymmetry of individual values is high and variation is low, and C is not very large. Figure 2.6 shows the BNE strategies for scenario 2 and the arrows in the four strategy squares represent the type contingent strategies of the four types of bidders. Please note this is an asymmetric pure strategy BNE in which T1 and T3 bidders have two alternative strategies, and the choice of them depends on N, L and C.

Figure 2.4 T2 bidders' Type Contingent Bids

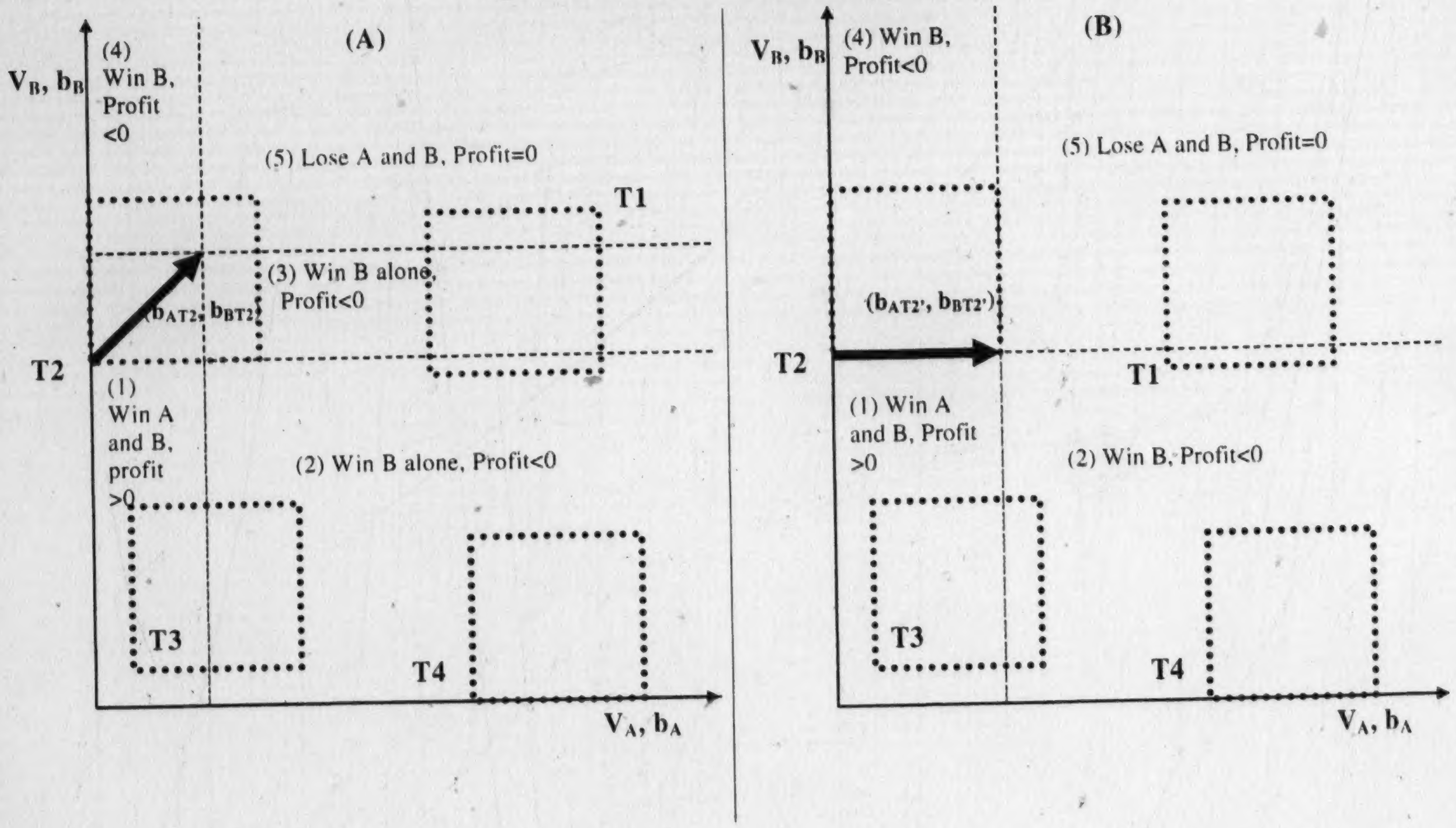
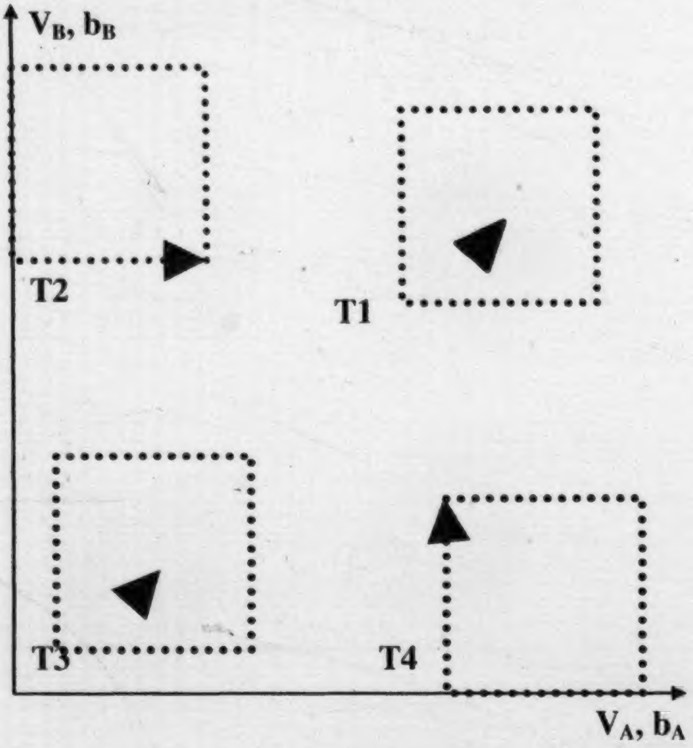

(A)
V_B, b_B
V_A, b_A
(4) Win B, Profit <0
(5) Lose A and B, Profit=0
(3) Win B alone, Profit<0
(b_{AT2}, b_{BT2})
(1) Win A and B, profit >0
(2) Win B alone, Profit<0
T1
T2
T3
T4
(B)
V_B, b_B
V_A, b_A
(4) Win B, Profit<0
(5) Lose A and B, Profit=0
$(b_{AT2'}, b_{BT2'})$
(1) Win A and B, Profit >0
(2) Win B, Profit<0
T1
T2
T3
T4

Figure 2.5 BNE Bids of the First Category



Similar to scenario 1, in the BNE, T2 and T4 bidders add all C to the bids of the low value product. They just bid their value for the higher value products. The reasons are again twofold, one is to avoid the exposure risk imposed by the bids from T1 bidders, and the other is try to outbid T3 bidders and win both A and B with a positive profit. The analysis is similar to that above.

What is different in the second category is that given T2 and T4 bidders' strategies, T1 bidder can not outbid both T2 and T4 and gain a profit, because now $(V_{AT4}+V_{BT2}) > (V_{AT1}+V_{BT1}+C)$. Therefore T1 bidders have to make a choice between two options (represented by the two smaller arrows in their strategy square). The first is to bid

(V_{AT4}, V_{BT2}), such that they outbid all T2, T3 and T4 bidders. This strategy leads to positive profits as long as T2 and T4 bidders do not exist simultaneously, as the T1 bidder can not outbid both and obtain a profit, since ($V_{AT4}+V_{BT2}$) > ($V_{AT1}+V_{BT1}+C$). The second strategy is to bid (V_{AT4}, 1+2L+C) to just try to outbid T3 and T4 bidders.[10] In the BNE, the number of T1 bidders that will adopt the first strategy depends on the magnitude of C, L and N, which decides the chance of winning both A and B with a profit and the chance of winning both A and B with a loss (See Appendix 2.4 for calculation).

In the BNE, T3 bidders also have two optional strategies, the first is to bid (-2L, V_{BT4}) and the second (V_{AT2}, V_{BT4}). Only one of the T3 bidders will choose the first and all others will choose the second strategy. The choice between the two strategies depends on the potential loss and gain of the two strategies for given C, L and N. The first strategy only results in a profit if all other bidders are of T3, and they choose the second strategy. However, if some of the other bidders are of T2, this strategy results in a loss because the T2 bidder will win B and the T3 bidder only wins A by overpaying. The second strategy for the T3 bidders is not subject to exposure risk since only when all her opponents are of T3 has she a positive chance to win both A and B with a profit.

In the third category are scenarios 6 and 8. For these two scenarios C is sufficiently large, such that there is also overlap between the T2 and T4 bidders' strategy squares. Figure 2.7 shows the BNE of scenario 6.

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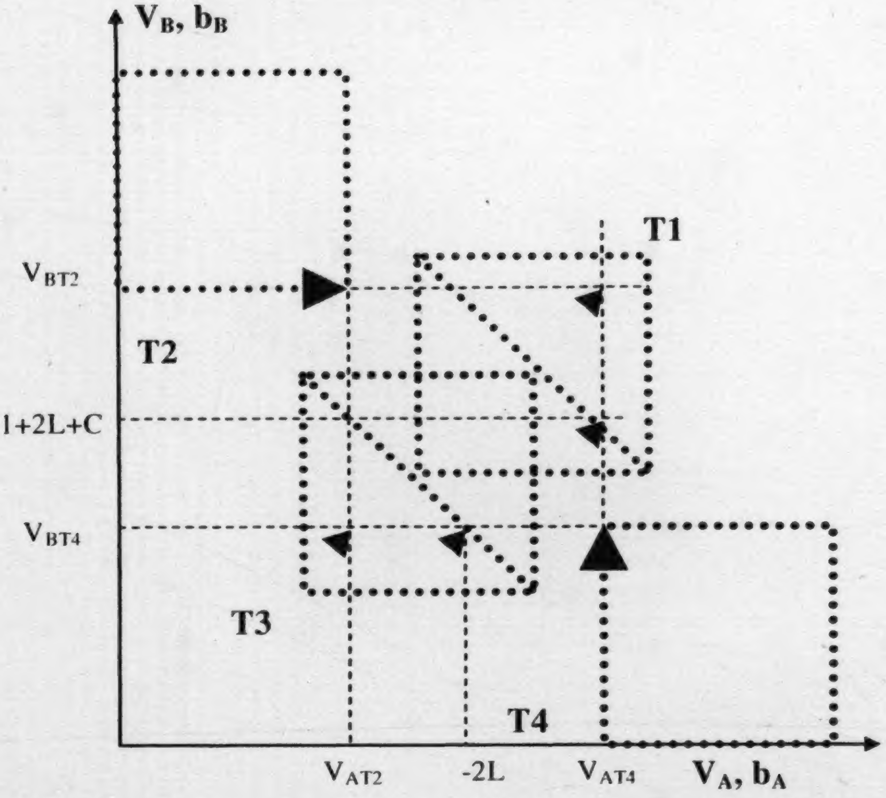
[10] Actually T1 bidders can also bid (1+2L+C, 1) to outbid T2 bidders. Since this strategy will not affect the revenue of separate auction, we don't discuss this strategy.

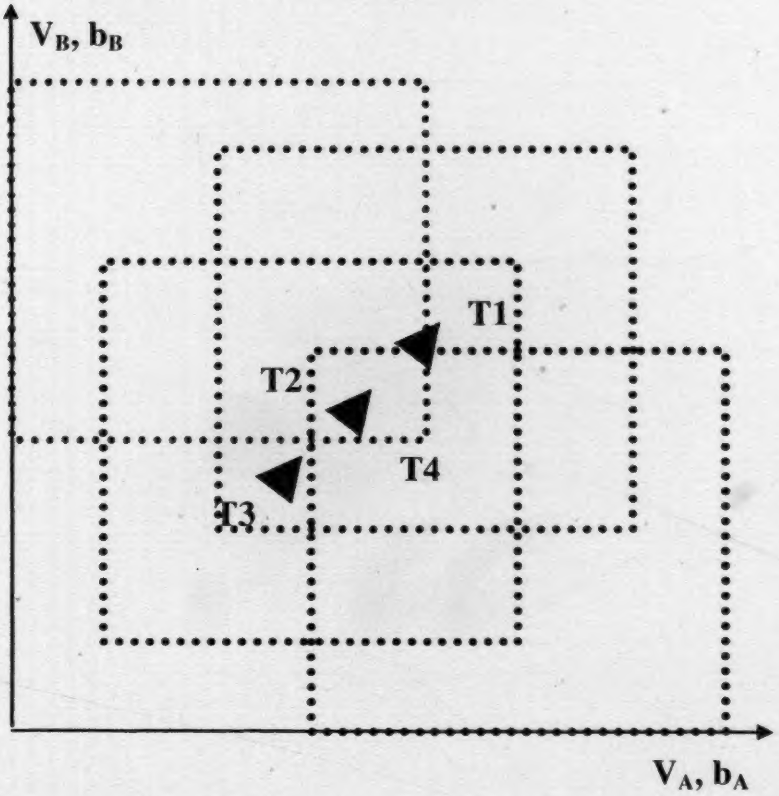
Figure 2.6 BNE of the Second Category



In the BNE, T2 and T4 bidders have the same type contingent bids (0.5+0.5C, 0.5+0.5C). They still add all C to the bids on the low value product, and T1 and T3 bidders add half of C to each of their two bids. We note that for scenario 6 and 8, when C is sufficiently large, neither asymmetry nor variation of individual values affects the BNE strategies.

The main difference between the BNE for the first category and the second category is that for category 2, T1 bidders cannot always safely outbid both T2 and T4 bidders and obtain a profit. This is due to smaller C (for a given level of L) or smaller L (for a given level of C) or both in category 2. In the third category's scenarios, C is larger than in the first category's scenarios, and therefore in the BNE T2 and T4 bidders' strategies merge since their strategy squares overlap.

Figure 2.7 BNE of the Third Category



2.3.3 Comparison of Revenues

Having the BNE bids of all four types of bidders for all the combinations of N, C and L for both bundled and separate auctions, we can calculate and compare the expected revenues of these two selling strategies (the calculations are provided in Appendix 2.5). Figure 2.9 shows the boundary conditions (defined by C and N) under which one bundled auction is at least as profitable as the two separate auction when L = 0.2112 or Rho= 0.5 (see Figure 2.8(A)), when L= 0 or Rho= 0 (see Figure 2.8(B)) and when L= -0.2112 or

Rho= -0.5 (see Figure 2.8(C)). The contours in Figure 2.8 are the ratios of the revenue of the two separate auctions to the revenue of a bundled auction for different combinations of C and N.

When N is small, bundled auctions are always more profitable. However, there is a threshold number of bidders (approximately three bidders), above which, separate auctions are at least as profitable as bundled auctions[11]. When N is larger than 3, for each L, there is a threshold amount of C, above which bundled auctions are at least as profitable as separate auctions. The threshold of C is approximately 0.3 when L= 0.221 (rho =0.5), C ≈ 0.6 when L= 0 (rho =0), and C ≈ 1.1 when L= -0.221 (rho= -0.5). Hence, when product values are negatively correlated, the magnitude of the complementarity has to be greater than half the value of the bundle for bundling to be more profitable.

As C increases, the difference between the revenues of the two selling mechanisms vanishes, and the two selling mechanisms become approximately equally profitable, since a larger C reduces the heterogeneity between V_A and V_B and, hence, the exposure risk.

How do the two dimensions of heterogeneity of V_A and V_B, variation and asymmetry, affect the ratio? To see the effect of asymmetry, we can compare Figure 2.8(A) (high variation/low asymmetry) versus Figure 2.8(B) (high variation/high asymmetry). High asymmetry of individual values not only increases the "more profitable" zone of separate auctions, but increases the difference in the revenues of the two selling mechanisms when separate auctions are more profitable. Comparing Figure

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[11] This is consistent with Palfrey (1983) and Chakraborty (1999) where complementarity is not present.

2.8(C) (low variation/ high asymmetry) with Figure 2.8(B), we see that high variation of individual values has exactly the opposite effect. It not only decreases the "more profitable" zone of separate auctions, but also decreases the difference in the revenues of the two selling mechanisms when separate auctions are more profitable.

According to these findings, an auctioneer of two complementary products should auction two products in a bundle when there are very few bidders attending the auction. When there are a large number of bidders, bundling should only be used when complementarity is sufficiently large. Therefore, if there is high asymmetry in individual values of the two products, only bundle them when complementarity is very large and/or when there are very few bidders (ex. C should be greater than about 1.1 when L= -0.2112/Rho=-0.5).

Figure 2.8(A) Low Asymmetry, High Variation (L=0.221, Rho=0.5)

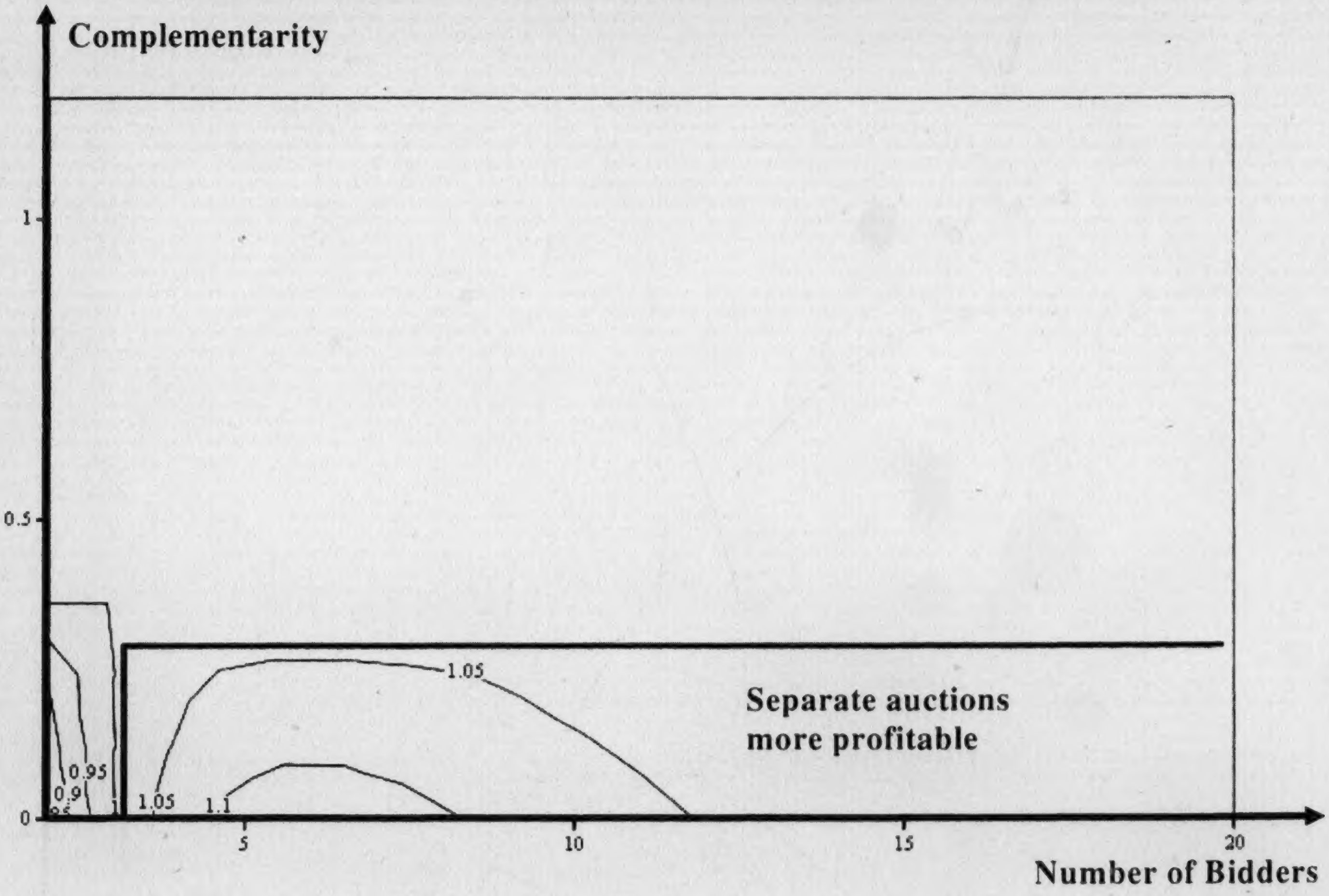


Figure 2.8(B) High Asymmetry, High Variation (L=0, Rho=0)

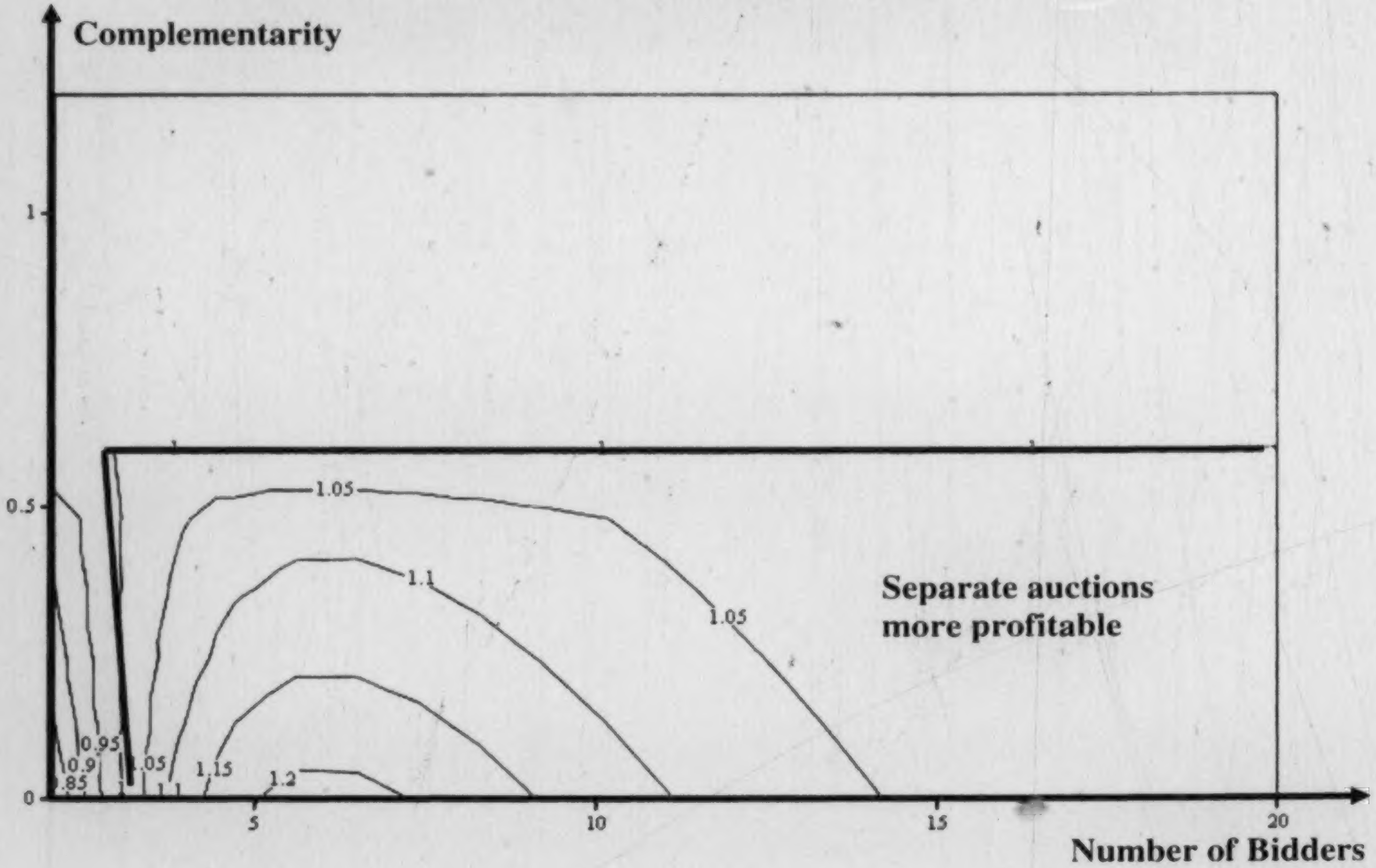
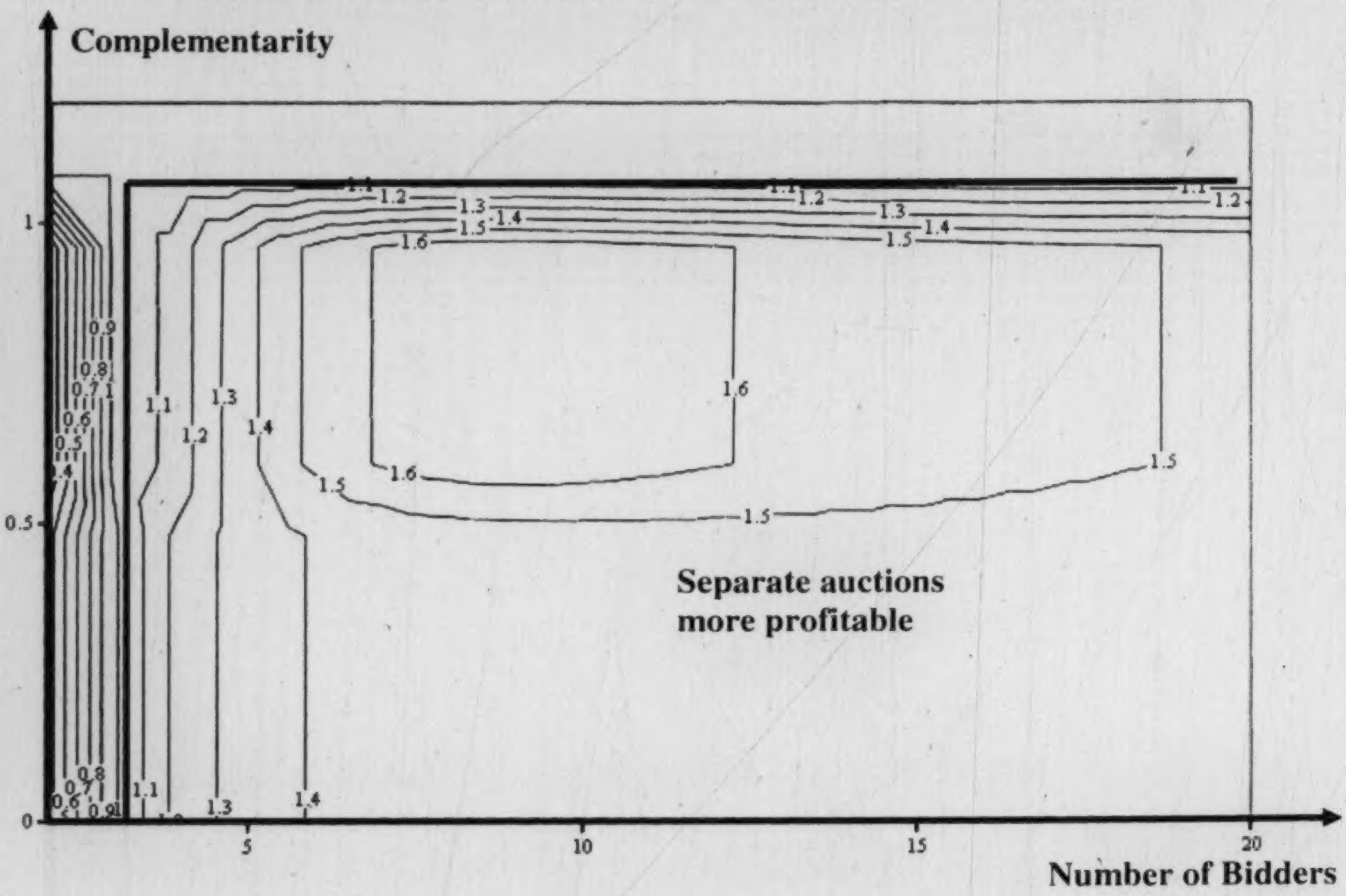


Figure 2.8(C) High Asymmetry, Low Variation(L=-0.221, Rho=-0.5)



2.4 Discussion

Profitability of separate auctions versus bundled auctions depends on the extent of inefficiency in bundled auctions and the degree of exposure risk in separate auctions. The relative profitability is measured through the ratio of revenue of separate auctions (say R_{se0}) to the revenue of bundled auctions (say R_{bu0}). When complementarity is zero there is no exposure risk in separate auctions, and the profitability only depends on the net effect of the inefficiency in bundled auction (see the bottom of Figure 2.9(A)-(C)). In all instances, regardless of asymmetry and variation, bundling is more profitable when there are only two bidders, and separate auctions become more profitable for three or more bidders.

When complementarity increases, bidders' bids in both bundled and separate auctions strictly increase, leading to higher revenues for both. However, this positive effect on revenue due to complementarity is unequal for the two mechanisms.

The bidding strategy in bundled auctions is simple. Bidders just add the entire complementarity to their bids for the bundle, regardless of L and N. Therefore, the revenue increases by as much as complementarity increases. In a certain sense, when $C > 0$, revenue of bundled auctions equals the sum of two components; the entire complementarity C and the revenues R_{bu0} without C which is affected by the inefficiency of bundling.

In separate auctions, depending on the extent of exposure risk, bidders will add an amount from zero to the entire C to each of their two bids (as Proposition 2 indicates). Therefore, in separate auctions, depending on N and L, the complementarity may increase the revenues by an amount ranging from zero to 2C. Like the revenue of bundled auctions, the revenue of separate auctions is also the sum of two different components: the revenue R_{se0} without C, which is only depended on L and N, and C^*, which is based on C and is a direct measurement of the exposure risk for given levels of L and N. Therefore the more that added to the revenue (i.e. greater C^*), the smaller the exposure risk.

Figure 2.9 shows the positive effect of C on the revenue of separate auctions. The contours show the proportion of C added to revenues (i.e. C^*/C) for different combinations of C, L, and N[12]. Figure 2.9(A), shows that C has the greatest positive effect on revenue when variation is high and asymmetry is low. This suggests that variation decreases the exposure risk.

Figure 2.9(B) shows, for high variation and high asymmetry, when C is less than about 0.5, and N is between 3 and 17, only part of C is added due to the exposure risk. Exposure risk is, in particular, caused by T2 and T4 bidders who add C to their bids for the lower value products. When N is greater than 17, there is a high likelihood that at least two T1 bidders are present who add the entire C to their bids. Since their bids decide revenues, the entire C is added to the revenue.

High asymmetry and low variation's effect on revenue is two fold (see Figure 2.9 (C)). When C is small (less than 0.5) and N is larger than 3, only part of C is added to the revenue, especially when N is large (>10). Under this condition separate auctions are more profitable than bundled auctions, because the extra C added to the revenue of the bundled auctions is not enough to offset the inefficiency of bundling. When C is between about 0.5 and 1 and N is small (<5), very interestingly, more than C is added to the revenue, since high value bidders run a very small exposure risk. T1 bidders add more than C to their bids to outbid both T2 and T4 bidders.

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[12] C^* is the difference between the revenues of separate auctions with C (see Appendix 2.5 for calculation) and the revenue of separate auctions without complementarity (see Appendix 2.1 for calculation)

The reason the model needs both asymmetry and variation is because the way N affects revenues depends not only on the correlation between V_A and V_B but also on the magnitude of the complementarity. Figure 2.9 clearly shows that in this model the heterogeneity of two values can not be represented sufficiently by the correlation of V_A and V_B and it is necessary to include both asymmetry and variation. For example, in Figure 2.9(B), the effect of N on the ratios of two revenues depends on the magnitude of complementarity C, which is standardized as is the distribution of V_A and V_B. Suppose we have two different distributions: distribution 1 with both high asymmetry and high variation and distribution 2 with both low asymmetry and low variation. In this case both distributions have the same correlation of V_A and V_B equal to zero. For a given amount of C, in the first distribution C is smaller after being standardized, say it equals 0.3. As N increases from 2, the ratio of the two revenues decreases first and then increases after N reaches 7. However, in the second distribution where the same C is greater after the standardization, say it is 1.5, N does not have much impact on the ratio.

So the study of relative profitability of bundled auctions versus separate auctions is essentially the study of the net effect of inefficiency in bundled auctions which award the products to bidders without the highest values and exposure risk in separate auctions which prevents bidder from bidding high when complementarity is present. When the latter has a greater impact, bundling is more profitable.

Figure 2.9(A) Effect of C on Revenue of Separate Auctions When L=0.2112/ Rho=0.5

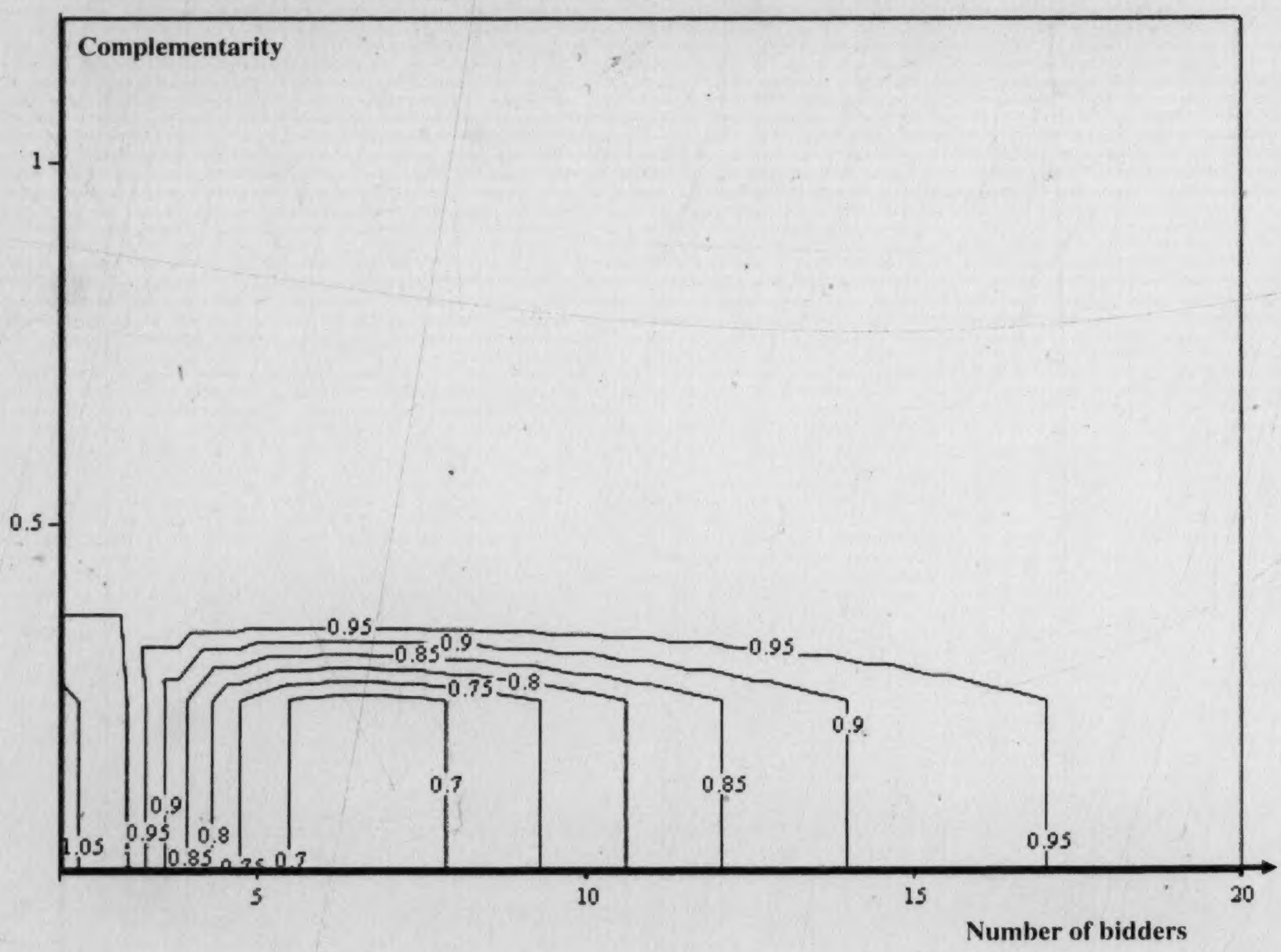


Figure 2.9(B) Effect of C on Revenue of Separate Auctions When L=0/ Rho=0

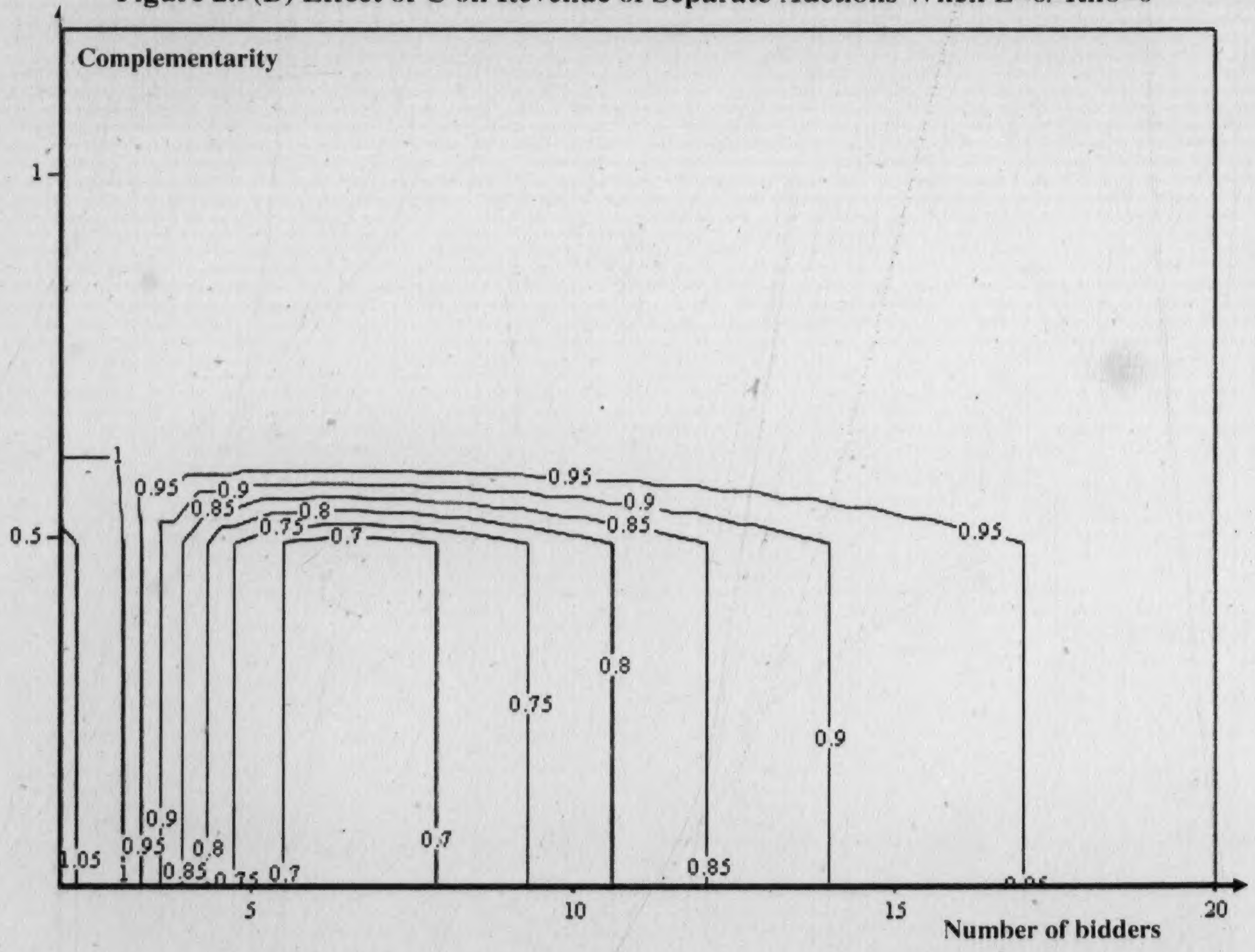
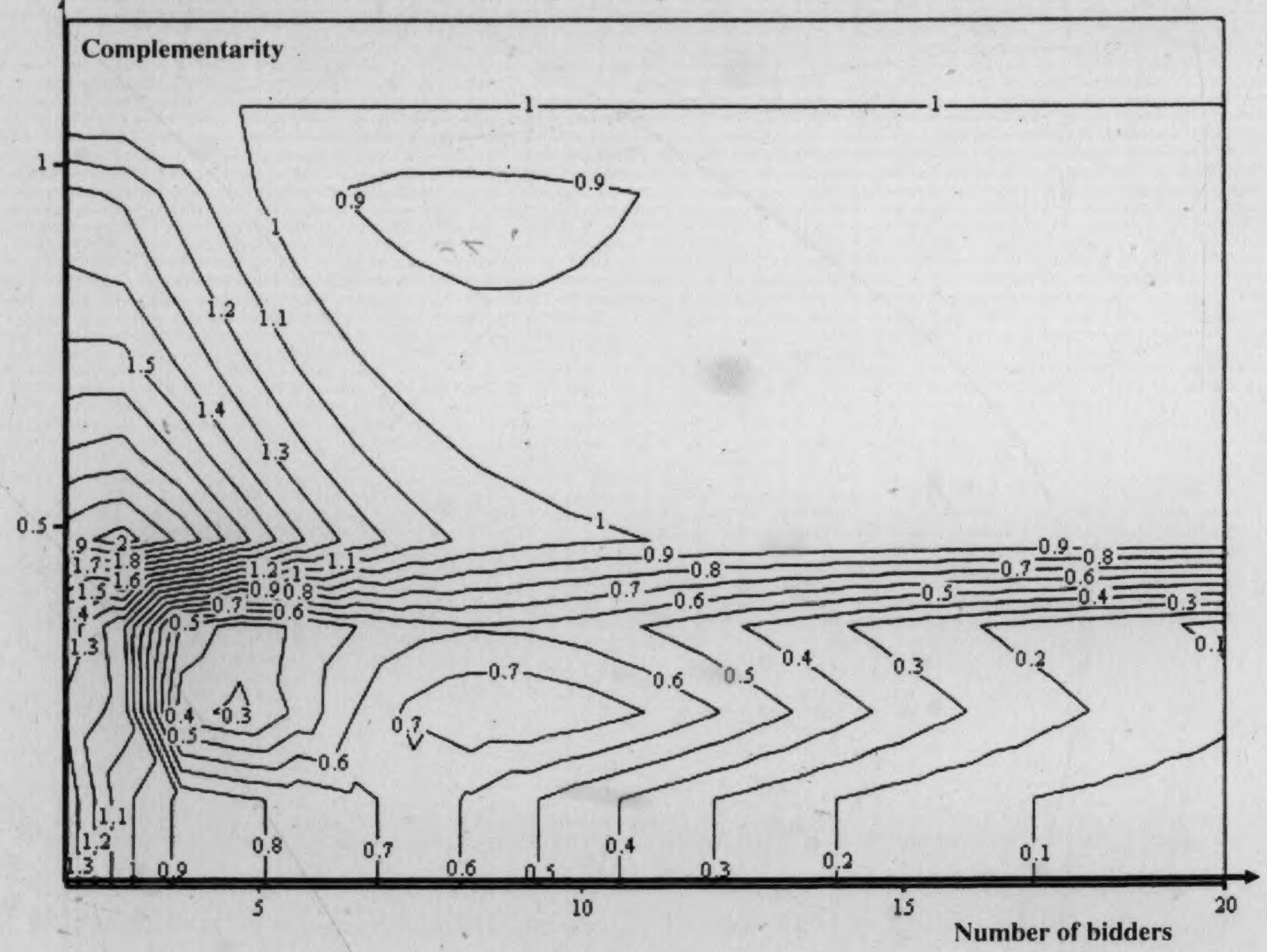


Figure 2.9(C) Effect of C on Revenue of Separate Auctions When L=-0.2112/ Rho=-0.5



Appendix 2.1: Revenue Calculation When C=0

When completmentarity is not present, in two separate auctions, each bidder's optimal strategy is to bid her individual values of product A and B. Therefore the revenues of separate auctions are essentially decided by bidders' values, i.e. the realization of the value distribution.

2.1.1 Revenue of Two Separate Auctions When $0 \le L \le 0.5$[13] ($0 \le Rho \le 1$)

When there are only 2 bidders, there are five potential combinations of two revenues in auction A and B, R33, R42, R22, R44, and R24 (see Figure 2.10(A) and Table 2.11(C) for total revenues). Figure 2.10 demonstrates all these revenues, where the small circles represent each type's strategy and the small triangles are the potential revenues. Table 2.11(A) shows how the likelihood of each potential combination is calculated. For example, for combination R33, when revenues of the two auctions are 0 and 0, either both bidders are of T3, or one bidder is of T3 and the other is one of the following types; T1, T2 and T4.

The expected revenue is equal to the weighted sum of all potential revenues. Therefore according to Table 2.11(A), when there are two bidders, the expected revenue for any given L of separate auctions is:

$R33\times(2\times0.25\times0.75+0.25\times0.25)+R42\times(2\times0.25\times0.25)+R22$[14]$\times(2\times0.25\times0.25+0.25\times0.25)\times2+R24\times(0.25\times0.25)$

$=0\times(2\times0.25\times0.75+0.25\times0.25)+2L\times(2\times0.25\times0.25)+1\times(2\times0.25\times0.25+0.25\times0.25)\times2+(2-2L)\times(0.25\times0.25)$

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[13] For the definition of L, please see page 18.

[14] R22 and R44 have the same likelihood. so we put them together.

Figure 2.10 Potential Revenues in Two Separate Auctions When L>0 and C=0

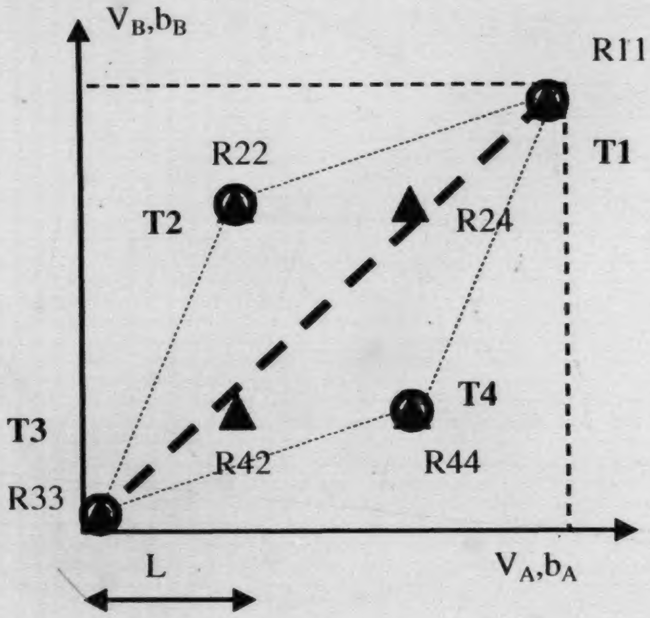


Table 2.5(A) Potential Revenues of Two Separate Auction and the Likelihoods When C=0, L>0, and N=2

Revenue	Likelihood	Explanation
R33	$2 \times 0.25 \times 0.75 + 0.25 \times 0.25$	One (T3), One (T1,T2,T4); Or Two (T3)*
R42	$2 \times 0.25 \times 0.25$	One (T2), One (T4)
R22,R44**	$(2 \times 0.25 \times 0.25 + 0.25 \times 0.25) \times 2$	For R22: One (T2), One (T1); Or Two (T2)
R24	0.25×0.25	Two (T1)

* One (T3) means one of the bidders is of Type 3; One (T1,T2,T4) Means one of the bidders is of one of Type 1,2 and 4; Two (T3) means two bidders are of Type 3.
** R22 and R44 have the same likelihood.

When there are more than two bidders, there is one more potential combination of revenues R11. Table 2.5(B) shows how the likelihoods of all potential revenues are calculated when N>2. For given N and L, the expected revenue is equal to the weighted sum of all potential revenues.

Table 2.5(B) Potential Revenues of Two Separate Auction and the Likelihoods When C=0, L>0, and N>2

Revenue	Likelihood	Explanation
R33	$N\times0.75\times0.25^{(N-1)}+0.25^{N}$	One (T1.T2.T4), Rest (T3); Or All (T3)
R42	$N\times0.25\times(N-1)\times0.25\times0.25^{(N-2)}$	One (T2). One (T4), Rest (T3)
R22. R44*	$\{[N\times0.25\times(N-1)\times0.25\times0.25^{(N-2)}$ $+N\times0.5\times[0.5^{(N-1)}-(N-1)\times0.25\times0.25^{(N-2)}-0.25^{(N-1)}]$ $+[0.5^{N}-N\times0.25\times0.25^{(N-1)}-0.25^{N}]]\}\times2$	For R24: One (T2). One (T1).Rest (T3): Or Two (T2). One (T1.T4). Rest (T2.T3); Or Two (T2), Rest (T2.T3)
R24	$N\times0.25\times[0.75^{(N-1)}-0.5^{(N-1)}-0.5^{(N-1)}+0.25^{(N-1)}]$ $+[0.75^{N}-0.5^{N}-0.5^{N}-N\times0.25\times0.5^{(N-1)}-N\times0.25\times0.5^{(N-1)}$ $+0.25^{N}+N\times0.25\times0.25^{(N-1)}+N\times0.25\times0.25^{(N-1)}+N\times0.25\times(N-1)\times0.25\times0.25^{(N-2)}]$	One (T1).One (T2).One (T4).Rest (T2.T3.T4): Or Two (T2).Two (T4).Rest (T2,T3,T4)
R11	$1-0.75^{N}-N\times0.25\times0.75^{(N-1)}$	Two (T1). Rest (All)**

* R22 and R44 have the same likelihood

** All but the two bidders are of any of the four types.

Table 2.5(C) Potential Revenues of Two Separate Auction When L>0 and C=0

Revenue	Amount
R33	0
R42	2L
R22,R44	1
R24	2-2L
R11	2

2.1.2 Revenue of Two Separate Auctions When -0.5 ≤ L ≤0 (-1 ≤ Rho ≤ 0)

Figure 2.11 demonstrates all the potential revenues, where the small triangles are potential revenues and small circles are type contingent strategies. Table 2.5(A) and 2.5(B) show how the likelihoods of all potential revenues are calculated when N=2 and N>2. Table 2.5(C) shows the amounts of all the potential revenues. Again for given N and L, the expected revenue is equal to the weighted sum of all potential revenues.

Figure 2.11 Potential Revenues in Two Separate Auctions When L<0 and C=0

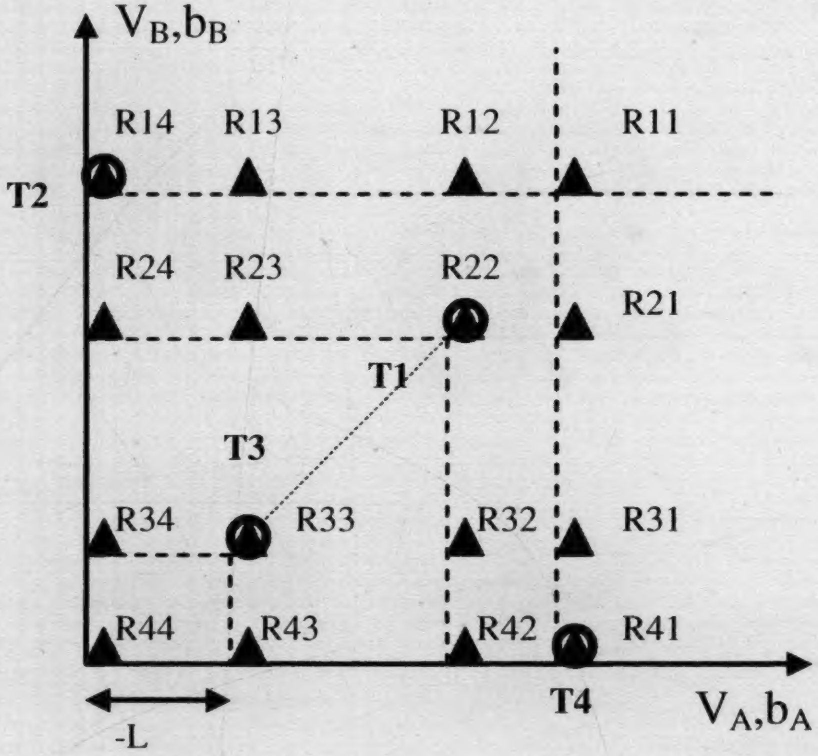

V_B,b_B
R14
R13
R12
R11
T2
R24
R23
R22
R21
T1
T3
R34
R33
R32
R31
R44
R43
R42
R41
T4
V_A,b_A
-L

Table 2.6(A) Potential Revenues of Two Separate Auction and the Likelihoods When C=0, L<0, and N=2

Revenue	Likelihood	Explanation
R44	$2\times0.25\times0.25$	One (T2), One (T4)
R34,R43*	$2\times0.25\times0.25\times2$	For R34: One (T2), One(T3)
R33	$0.5^2-0.25^2$	One (T3), One (T1,T3)
R24.R42*	$2\times0.25\times0.25\times2$	For R24: One (T1), One (T2),
R14,R41*	$0.25\times0.25\times2$	For R14: Two (T2)
R22	0.25×0.25	Two (T1)

Note: *Have the same likelihood.

Table 2.6(B) Potential Revenues of Two Separate Auction and the Likelihoods When C=0, L<0, and N>2

Revenue	Likelihood	Explanation
R33	$N\times0.25\times(N-1)\times0.25\times0.25^{(N-2)}$ $+ N\times0.75\times0.25^{(N-1)}$ $+ 0.25^{N}$	One (T2). One (T4). Rest (T3): Or One (T1.T2.T4). Rest (T3): Or All (T3)
R23.R32*	$N\times0.25\times(N-1)\times0.25\times0.25^{(N-2)}\times2$	For R23: One(T1). One (T2). Rest (T3)
R14.R41*	$(N\times0.75\times0.25^{(N-1)}$ $+ 0.25^{N})\times2$	For R14: One (T1.T3.T4). Rest(T2): Or All (T2)
R22	$N\times0.25\times(N-1)\times0.25\times[0.5^{(N-2)}-0.25^{(N-2)}]$ $+ N\times0.25\times[0.5^{(N-1)}-0.25^{(N-1)} -(N-1)\times0.25\times0.25^{(N-2)}]$ $+ N\times0.25\times[0.5^{(N-1)}-0.25^{(N-1)} -(N-1)\times0.25\times0.25^{(N-2)}]$ $+ [0.5^{N}-0.25^{N} -N\times0.25\times0.25^{(N-1)}]$	One(T2). One (T4). One (T1). rest (T1.T3): Or One(T2). Two(T1). Rest (T1.T3): Or One(T4). Two(T1). Rest (T1.T3): Or Two(T1). Rest (T1.T3)
R13.R31*	$\{[0.5^{N}-0.25^{N}-0.25^{N}-N\times0.25\times0.25^{(N-1)} - N\times0.25\times0.25^{(N-1)}]$ $+N\times0.5\times[0.5^{(N-1)}-0.25^{(N-1)} -0.25^{(N-1)}-(N-1)\times0.25\times0.25^{(N-2)}]\}\times2$	For R13: Two (T2). Two (T3). Rest (T2.T3): Or Two (T2), One (T3). One (T1,T4), Rest (T2.T3)

Table 2.6(B) Potential Revenues of Two Separate Auction and the Likelihoods When C=0, L<0, and N>2 (continued)

R12,R21*	$\{[0.75^N-0.5^N-0.5^N-N\times0.25\times0.5^{(N-1)}-N\times0.25\times0.5^{(N-1)}+0.25^N+N\times0.25\times0.25^{(N-1)}+N\times0.25\times0.25^{(N-1)}+N\times0.25\times(N-1)\times0.25\times0.25^{(N-2)}]$ $+N\times0.25\times[0.75^{N-1}-0.5^{N-1}-0.5^{N-1}-(N-1)\times0.25\times0.5^{(N-2)}+0.25^{N-1}+(N-1)\times0.25\times0.25^{(N-2)}]\}\times2$	For R12: Two (T1). Two (T2). Rest (T1.T2.T3): Or One (T4).Two (T2). One(T1). Rest (T1.T2.T3)
R11	$[1-0.75^N-0.75^N-N\times0.25\times0.75^{(N-1)}-N\times0.25\times0.75^{(N-1)}+0.5^N+N\times0.25\times0.5^{(N-1)}+N\times0.25\times0.5^{(N-1)}+N\times0.25\times(N-1)\times0.25\times0.5^{(N-2)}]$	Two (T2). Two (T4). Rest (all)

*Have the same likelihood

Table 2.6(C) Potential Revenues of Two Separate Auction When L<0 and C=0

Revenue	Amount
R44	0
R34,R43	-L
R33	-2L
R24,R42	1+L
R23,R32	1
R14,R41	1
R22	2+2L
R13,R31	1-L
R12,R21	2+L
R11	2

2.1.3 Revenue for One Auction for the Bundle When C=0

In one bundled auction, a bidder's optimal strategy is to bid her sum of the two individual values. As Figure 2.12 shows, there are only three potential revenues (since T2 and T4 bidders have the same sum for the two values). Table 2.6(A) and 2.6(B) show how the likelihoods of all potential revenues are calculated when N=2 and N>2. Table 2.6(C) shows the amounts of all the potential revenues. Again, for given N and L, the expected revenue is equal to the weighted sum of all potential revenues.

Figure 2.12 Potential Revenues in One Bundled Auction When C=0

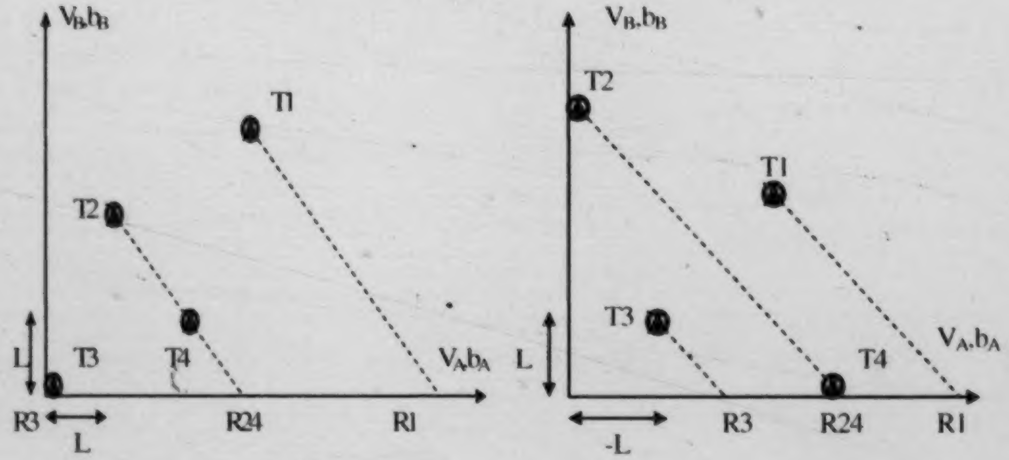


Table 2.7(A) Potential Revenues of One Bundled Auction and the Likelihoods When C=0 and N=2

Revenue	Likelihood	Explanation
R1	0.25×0.25	Two (T1)
R24	$2\times0.5\times0.25$ $+0.5^2$	One (T2,T4), One (T1); Or Two (T2,T4)
R3	$2\times0.75\times0.25$ $+0.25^2$	One (T1,T2,T4), One (T3); Or Two (T3)

Table 2.7(B) Potential Revenues of One Bundled Auction and the Likelihoods When C=0 and N>2

Revenue	Likelihood	Explanation
R1	$1-0.75^{N}-N\times0.25\times0.75^{(N-1)}$	Two (T1), Rest (ALL)
R24	$N\times0.25\times[0.75^{(N-1)}-0.25^{(N-1)}]$ $+[0.75^{N}-0.25^{N}-N\times0.5\times0.25^{(N-1)}]$	One (T2,T4), One (T1), Rest (T2,T3,T4); Or Two (T2,T4), Rest (T2,T3,T4)
R3	$N\times0.75\times0.25^{(N-1)}$ $+0.25^{N}$	One (T1,T2,T4), Rest (T3); Or All (T3)

Table 2.7(C) Potential Revenues of One Bundled Auction When C=0

Revenue	Amount (L>0)	Amount (L<0)
R1	2	2+2L
R24	1	1
R3	0	-2L

Appendix 2.2: Weakly Dominant Strategies in Two Simultaneous Separate Auctions

Result 2: *in two simultaneous separate auctions, bidder i's (i=1, 2,..., N) weakly dominant bids satisfy*

$$V_{Ai} \leq b_{Ai} \leq V_{Ai} + C,$$
$$V_{Bi} \leq b_{Bi} \leq V_{Bi} + C,$$

where V_{Ai} *,* V_{Bi} *are bidder i's individual values of A and B; C is the value of complementarity, and* b_{Ai} *,* b_{Bi} *are the two bids of bidder i in the two separate auctions.*

In other words, for each strategy outside the strategy square, there is a strategy inside the square which yields at least as much profit.

Proof:

First we want to prove that for each strategy (b_{A1}, b_{B1}, denoted strategy 1) outside the square, there is a strategy inside the square, say strategy 2, that is at least as profitable.

Suppose the highest b_A' and b_B from the remaining N-1 bidders are respectively b_{Ah} and b_{Bh} (not necessarily from the same bidder) and there are j ($1 \leq j \leq N-1$) bidders who bid b_{Ah} and k ($1 \leq k \leq N-1$) bidders who bid b_{Bh}. Let us consider strategy 1, and examine the possible bids outside the square.

Here is how the profits are determined when bidder *i* chooses strategy 1 (b_{A1}, b_{B1}).

- When $b_{Ah} \geq b_{A1}$ and $b_{Bh} > b_{B1}$, or $b_{Ah} > b_{A1}$ and $b_{Bh} \geq b_{B1}$, her profit is zero;
- When $b_{Ah} < b_{A1}$ and $b_{Bh} \leq b_{B1}$, or $b_{Ah} \leq b_{A1}$ and $b_{Bh} < b_{B1}$, her profit is ($V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}$);
- When $b_{Ah} < b_{A1}$ and $b_{Bh} > b_{B1}$, her profit is ($V_{Ai} - b_{Ah}$);
- When $b_{Ah} > b_{A1}$ and $b_{Bh} < b_{B1}$, her profit is ($V_{Bi} - b_{Bh}$);

- When $b_{Ah}=b_{A1}$ and $b_{Bh}=b_{B1}$, her profit is $1/[(1+\min(j, k)]\ (V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh})$ < 0, where min(j, k) is the minimum of j and k;

The calculation of profit for strategy 2 is exactly the same as that of strategy 1.

Now let us compare the profits of strategy 1 and 2 for each of the five possibilities.

(1) When $V_{Ai}+ C < b_{A1}$, $V_{Bi}+ C < b_{B1}$, strategy 1 (b_{A1}, b_{B1}) is weakly dominated by strategy 2 ($V_{Ai}+ C$, $V_{Bi}+ C$).

Table 2.8 compares the profits of strategies 1 and 2 for all combinations of b_{Ah} (column) and b_{Bh} (row). In each cell, the first row shows the profit of strategy 1 and the second the profits for strategy 2, for a given combination of b_{Ah} and b_{Bh}. We can see that for each of the possible 25 combinations of b_{Ah} and b_{Bh}, strategy 2 is at least as profitable as strategy 1, so strategy 2 weakly dominates strategy 1 (b_{A1}, b_{B1}).

Table 2.8 Comparisons of Profits of Strategy 1 and 2 When $V_{Ai}+ C < b_{Al}$ and $V_{Bi}+ C < b_{Bl}$

	$b_{Ah} > b_{Al}$	$b_{Ah} = b_{Al}$	$b_{Al}>b_{Ah} > V_{Ai}+ C$	$b_{Ah} = V_{Ai}+ C$	$b_{Ah} < V_{Ai}+ C$
$b_{Bh} > b_{Bl}$	1: [0] 2: [0]	1: [0] 2: [0]	1: [V_{Ai} - b_{Ah}] 2: [0]	1: [-C] 2: [0]	1: [V_{Ai} - b_{Ah}] 2: [V_{Ai} - b_{Ah}]
$b_{Bh} = b_{Bl}$	1: [0] 2: [0]	1: [*] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [0]	1: [V_{Bi} + C- b_{Bh}] 2: [0]	1: [V_{Ai} - b_{Ah}] 2: [V_{Ai} - b_{Ah}]
$b_{Bl}>b_{Bh} > V_{Bi}+ C$	1: [V_{Bi} - b_{Bh}] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [V_{Ai} - b_{Ah}]
$b_{Bh} = V_{Bi}+ C$	1: [-C] 2: [0]	1: [V_{Ai}+C- b_{Ah})] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [0]	1: [0] 2: [0]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}]
$b_{Bh} < V_{Bi}+ C$	1: [V_{Bi} - b_{Bh}] 2: [V_{Bi} - b_{Bh}]	1: [V_{Bi} - b_{Bh}] 2: [V_{Bi} - b_{Bh}]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [V_{Bi} - b_{Bh}]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}]	1: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}] 2: [V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}]

* $1/[(1+\min(j, k)]\ (V_{Ai}+ V_{Bi} +C- b_{Ah}- b_{Bh}) < 0$, where min(j, k) is the minimum of j and k.

(2) When $V_{Bi} + C \geq b_{B1} \geq V_{Bi}$, $b_{A1} \geq V_{Ai} + C$, strategy 1 (b_{A1}, b_{B1}) is weakly dominated by strategy 2 (V_{A1}+C, b_{B1}).

Strategy 1 and 2 will lead to the same outcome in the auction for product B. Suppose bidder *i* does not win B. When $b_{Ah} < V_{Ai}$+ C, or $b_{Ah} > b_{A1}$, the two strategies lead to the same marginal profits for product A. When b_{A1}=b_{Ah}. strategy 1 leads to expected marginal profits of (V_{Ai} – b_{Ah}) which is negative, while strategy 2 leads to expected marginal profits of zero in auction A. When V_{Ai}+ C <b_{Ah}< b_{A1}. strategy 1 leads to expected marginal profit of (V_{Ai} –b_{Ah}) which is negative, while strategy 2 leads to expected marginal profit of zero in auction A. When V_{Ai}+ C =b_{Ah}. strategy 1 leads to expected marginal profit of (V_{Ai} –b_{Ah}) while strategy 2 also leads to expected profit of (V_{Ai} –b_{Ah}) in auction A.

If bidder *i* wins B, all (V_{Ai} –b_{Ah}) above should be replaced by (V_{Ai} –b_{Ah}+ C) and strategy 2 is still at least as good as strategy 1 (please note $V_{Ai} - b_{Ah} + C \leq 0$ in all cases). Therefore strategy 2 weakly dominates strategy 1 which is outside the square.

When $V_{Ai} + C \geq b_{A1} \geq V_{Ai}$ and $b_{B1} \geq V_{Bi} + C$, the proof is the same.

(3) When $b_{A1} \geq V_{Ai}$+C, $V_{Bi} > b_{B1}$, strategy 1 (b_{A1}, b_{B1}) is weakly dominated by strategy 2 (V_{Ai} + C, V_{Bi}) in the square.

Table 2 shows the profits of strategies 1 and 2 at all combinations of b_{Ah} and b_{Bh}. For each of all possible 25 combinations of b_{Ah} and b_{Bh}, strategy 2 is at least as profitable as strategy 1, so strategy 2 in the square weakly dominates strategy 1.

When $b_{B1} \geq V_{Bi}$+C, $V_{Ai} > b_{A1}$, the proof is the same.

Table 2.9 Comparisons of Profits of Strategy 1 and 2 When $V_{Bi} + C \geq b_{Bl} \geq V_{Bi}$ and $b_{Al} \geq V_{Ai} + C$

	$b_{Ah} > b_{Al}$	$b_{Ah} = b_{Al}$	$b_{Al} > b_{Ah} > V_{Ai} + C$	$b_{Ah} = V_{Ai} + C$	$b_{Ah} < V_{Ai} + C$
$b_{Bh} > V_{Bi}$	1: [0] 2: [0]	1: [0] 2: [0]	1:[0] 2:[V_{Ai} - b_{Ah}]	1:[-C] 2:[0]	1:[V_{Ai} - b_{Ah}] 2:[V_{Ai} - b_{Ah}]
$b_{Bh} = V_{Bi}$	1:[0] 2:[0]	1:[0] 2:[0]	1:[V_{Ai} - b_{Ah}] 2:[0]	1:[-C] 2:[0]	1:[V_{Ai} - b_{Ah}] 2:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}]
$V_{Bi} > b_{Bh} > b_{Bl}$	1:[0] 2:[V_{Bi} – b_{Bh}]	1:[0] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} - b_{Ah}] 2:[V_{Bi} – b_{Bh}]	1:[-C] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} + V_{Bi}] 2:[V_{Ai} - b_{Ah} +C- b_{Ah} - b_{Bh}]
$b_{Bh} = b_{Bl}$	1:[0] 2:[V_{Bi} – b_{Bh}]	1:[*] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}] 2:[V_{Bi} – b_{Bh}]	1:[-C] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}] 2:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}]
$b_{Bh} < b_{Bl}$	1:[V_{Bi} – b_{Bh}] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}] 2:[V_{Bi} – b_{Bh}]	1:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}] 2:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}]	1:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}] 2:[V_{Ai} + V_{Bi} +C- b_{Ah} - b_{Bh}]

* $1/[1+\min(j, k)]\,[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}] < 1/[1+\min(j, k)](V_{Ai} + C - b_{Ah}) + 1/[1+\min(j, k)](V_{Bi} - b_{Bh}) < [V_{Bi} - b_{Bh}]$, where min (j, k) is the minimum of j and k.

(4) When $V_{Bi} > b_{B1}$, $V_{Ai} + C \geq b_{A1} \geq V_{Ai}$, strategy 1 (b_{A1}, b_{B1}) is weakly dominated by strategy 2 (b_{A1}, V_{Bi}).

Strategy 1 and 2 lead to the same outcome for auction A. Suppose bidder *i* does not win A. When $b_{Bh} < V_{Bi} + C$, or $b_{Bh} > V_{Bi}$, both strategies lead to the same marginal profit for B. When $b_{B1} = b_{Bh}$, strategy 1 leads to expected marginal profit of $1/(j+1)(V_{Bi} - b_{Bh})$, while strategy 2 leads to expected marginal profit of $(V_{Bi} - b_{Bh})$ in auction B. When $b_{B1} < b_{Bh} < V_{Bi}$, strategy 1 leads to expected marginal profit of zero, while strategy 2 leads to expected marginal profit of $(V_{Bi} - b_{Bh})$ in auction A. When $b_{Bh} = V_{Bi}$, strategy 1 and 2 lead to expected marginal profit of zero in auction B. If the bidder *i* wins A, all $(V_{Bi} - b_{Bh})$ above should be replaced by $(V_{Bi} - b_{Bh} + C)$ and strategy 2 is still at least as good as strategy 1 in auction B. Therefore strategy 2 weakly dominates strategy 1 which is outside the square.

When $V_{Ai} > b_{A1}$, $V_{Bi} + C \geq b_{B1} \geq V_{Bi}$, the proof is same.

(5) When $V_{Ai} > b_{A1}$, $V_{Bi} > b_{B1}$, strategy 1 (b_{A1}, b_{B1}) is weakly dominated by strategy 2 (V_{Ai}, V_{Bi}).

The following Table 2.10 shows the profits of strategies 1 and 2 for all combinations of b_{Ah} and b_{Bh}. For each of the possible 25 combinations of b_{Ah} and b_{Bh}, strategy 2 is at least as profitable as strategy 1, so strategy 2 weakly dominates strategy 1.

Table 2.10 Comparisons of Profits of Strategy 1 and 2 When $V_{Ai}>b_{Al}$ and $V_{Bi}>b_{Bl}$

	$b_{Ah} > V_{Ai}$	$b_{Ah} = V_{Ai}$	$V_{Ai} > b_{Ah} > b_{Al}$	$b_{Ah} = b_{Al}$	$b_{Ah} < b_{Al}$
$b_{Bh} > V_{Bi}$	1: [0] 2: [0]	1: [0] 2: [0]	1: [0] 2: $[V_{Ai} - b_{Ah}]$	1: [0] 2: $[V_{Ai} - b_{Ah}]$	1: $[V_{Ai} - b_{Ah}]$ 2: $[V_{Ai} - b_{Ah}]$
$b_{Bh} = V_{Bi}$	1: [0] 2: [0]	1: [0] 2: [*]	1:[0] 2:$[V_{Ai} - b_{Ah}]$	1:[0] 2:$[V_{Ai} - b_{Ah}]$	1:$[V_{Ai} - b_{Ah}]$ 2:$[V_{Ai} - b_{Ah}]$
$V_{Bi} > b_{Bh} > b_{Bl}$	1: [0] 2: $[V_{Bi} - b_{Bh}]$	1:[0] 2:$[V_{Bi} - b_{Bh}]$	1:[0] 2:$[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$	1:[0] 2:$[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$	1:$[V_{Ai} - b_{Ah}]$ 2:$[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$
$b_{Bh} = b_{Bl}$	1: [0] 2: $[V_{Bi} - b_{Bh}]$	1:[0] 2:$[V_{Bi} - b_{Bh}]$	1:[0] 2:$[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$	1:[**] 2:$[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$	1: $[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$ 2: $[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$
$b_{Bh} < b_{Bl}$	1: $[V_{Ai} - b_{Ah}]$ 2: $[V_{Ai} - b_{Ah}]$	1:$[V_{Ai} - b_{Ah}]$ 2:$[V_{Ai} - b_{Ah}]$	1:$[V_{Bi} - b_{Bh}]$ 2:$[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$	1: $[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$ 2: $[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$	1: $[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$ 2: $[V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh}]$

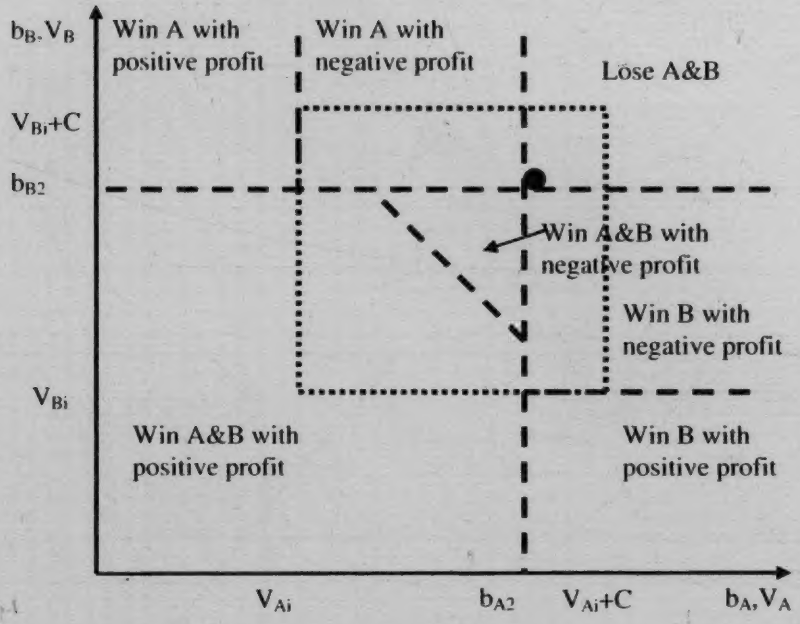
* $1/[1+\min(j, k)]C$, where min (j, k) is the minimum of j and k.

** $1/[1+\min(j, k)](V_{Ai} + V_{Bi} + C - b_{Ah} - b_{Bh})$, where min (j, k) is the minimum of j and k.

Second, we would like to prove that no strategy in the square weakly dominates another strategy in the square.

Table 2.10 shows strategy 2's payoffs for all possible locations of (b_{Ah}, b_{Bh}). Figure 2.13 gives a graphic demonstration of how the location of (b_{Ah}, b_{Bh}) decides strategy 2's profit. To prove that there is no strategy in the square that is always as least as profitable as another one in the square, we just need to show any change of strategy 2 (which leads to a new strategy inside the square) can not make bidder i always better off for all possible (b_{Ah}, b_{Bh}).

Figure 2.13 Profitability of a Strategy in the Strategy Square


b_B, V_B
Win A with positive profit
Win A with negative profit
Lose A&B
$V_{Bi}+C$
b_{B2}
Win A&B with negative profit
Win B with negative profit
V_{Bi}
Win A&B with positive profit
Win B with positive profit
V_{Ai}
b_{A2}
$V_{Ai}+C$
b_A, V_A

Depending on (b_{Ah}, b_{Bh}), strategy 2 can win one or both products with positive or negative profit (only when $V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}>0$, bidder i can get both products with a profit). Now we can see no matter how we change strategy 2, we cannot increase the "profit" zones without increasing the "loss" zone. If we increase b_{B2}, we not only increase the "Win A with positive profit" and "Win A and B with positive profit" zones, but increase the "Win B with negative profit" zone. In other words, we can not come up with a new strategy that which always generates more profit than strategy 1. So we prove that the square is the set for all the strategies which are not weakly dominated.

Table 2.11 Profits of Strategy 2 in the Strategy Square

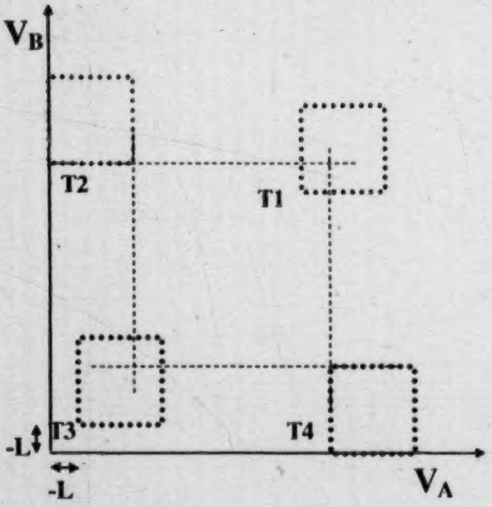
	$b_{Ah}>b_{Al}$	$b_{Ah}=b_{Al}$	$V_{Ai}<b_{Ah}<b_{Al}$	$b_{Ah}=V_{Ai}$	$b_{Ah}<V_{Ai}$
$b_{Bh}>b_{Bl}$	0	0	$(V_{Ai}-b_{Ah})$	0	$(V_{Ai}-b_{Ah})$
$b_{Bh}=b_{Bl}$	0	*	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$
$V_{Bi}<b_{Bh}<b_{Bl}$	$(V_{Bi}-b_{Bh})$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Bi}+C-b_{Bh}]$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$
$b_{Bh}=V_{Bi}$	0	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Ai}+C-b_{Ah}]$	C	$[V_{Ai}+C-b_{Ah}]$
$b_{Bh}<V_{Bi}$	$(V_{Bi}-b_{Bh})$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$	$[V_{Bi}+C-b_{Bh}]$	$[V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh}]$

* $1/[1+\min(j,k)](V_{Ai}+V_{Bi}+C-b_{Ah}-b_{Bh})$, where min (j, k) is the minimum of j and k.

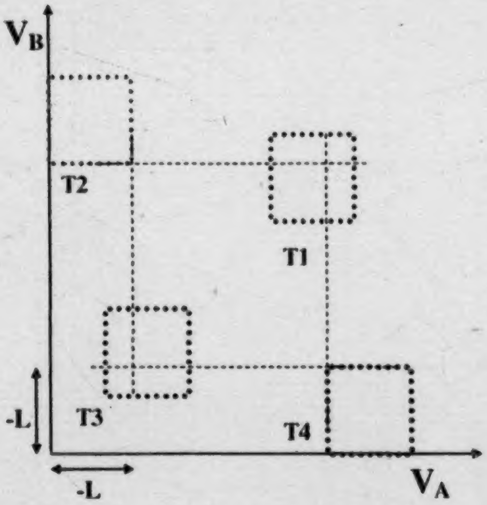
Appendix 2.3: All Scenarios

Figure 2.14 (A) When L (Rho) ≤ 0

Scenario 1 : when $0 \le -L \le 0.5C$,$0 \le C \le 0.5$

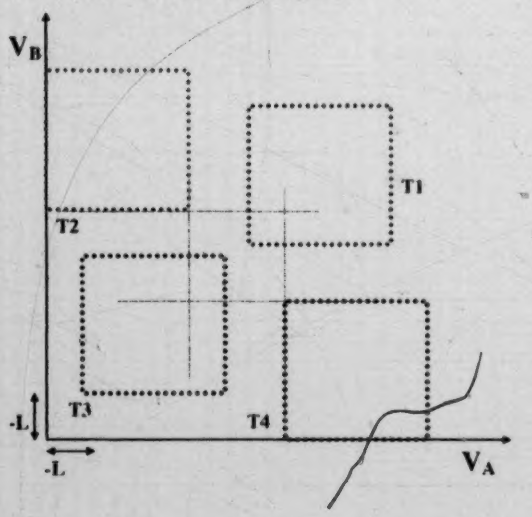

V_B
T2
T1
$-L$
T3
T4
$-L$
V_A

Scenario 2: when $0.5C \le -L \le C$, $0 \le C \le 0.5$


V_B
T2
T1
$-L$
T3
T4
$-L$
V_A

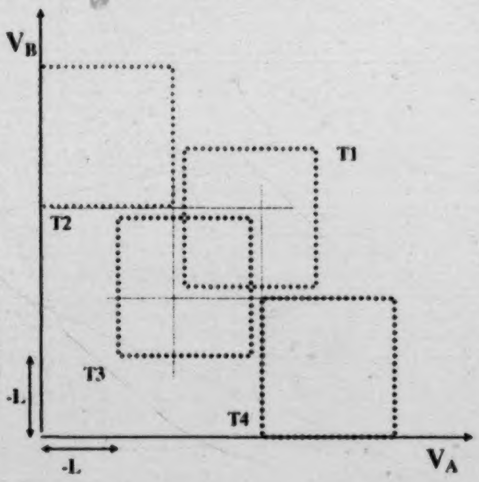
Scenario 3: $C \le -L \le 0.5$, $0 \le C \le 0.5$;

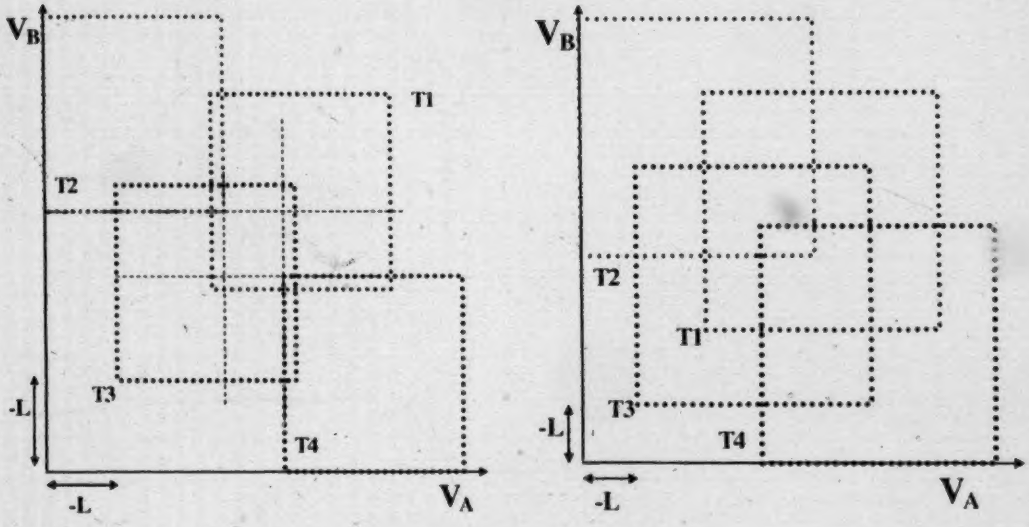
Or $0 \le -L \le 0.5C$, $0.5 \le C \le 2/3$;


V_B
T1
T2
$-L$
T3
T4
$-L$
V_A

Scenario 4: $0.5C \le -L \le 0.5$, $0.5 \le C \le 2/3$

Or $0 \le -L \le 0.5C$, $2/3 \le C \le 1$


V_B
T1
T2
$-L$
T3
T4
$-L$
V_A

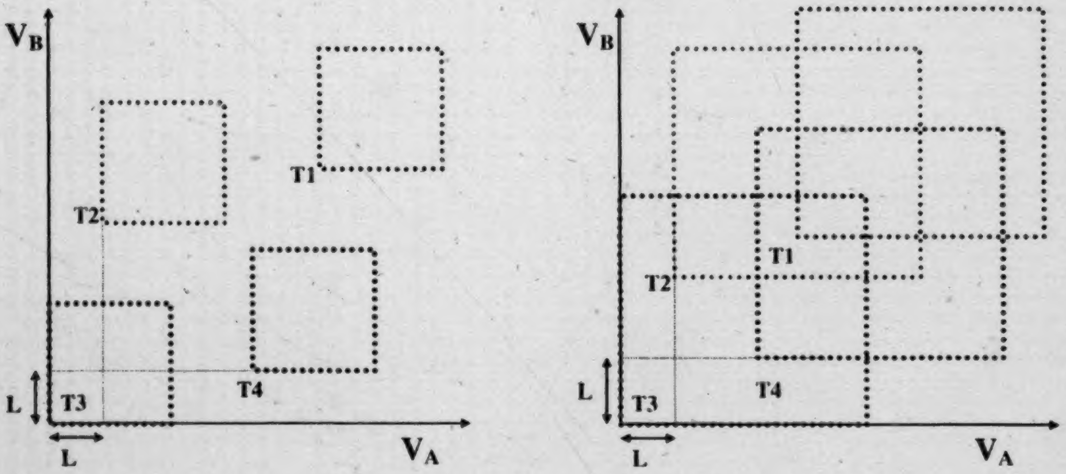


Scenario 5: 0.5C≤ -L ≤0.5, 2/3≤ C ≤1

Scenario 6: 0≤ -L ≤0.5, 1≤ C ≤ +∝

V_B
T1
T2
T3
T4
-L
-L
V_A
V_B
T2
T1
T3
T4
-L
-L
V_A

Figure 2.14 (B) When L (Rho) >0



Scenario 7: 0≤ L ≤ 0.5, 0≤ C ≤0.5-L

Scenario 8: 0≤ L ≤ 0.5, (0.5-L) ≤ C ≤ +∞

V_B
T1
T2
T4
L
T3
L
V_A
V_B
T1
T2
L
T3
T4
L
V_A

Appendix 2.4: Proof of Bayesian Nash Equilibria (BNE) in Two Simultaneous Separate Vickrey Auctions

Theorem 1 (Equilibrium Statement): *for two separate auction of N bidders, in scenario 1, 3, 5 and 7, a BNE will consist of four type-contingent strategies.*

Types	b_A	b_B
T1	$V_{AT1}+0.5C$	$V_{BT1}+0.5C$
T2	$V_{AT2}+C$	V_{BT2}
T3	$V_{AT3}+0.5C$	$V_{BT3}+0.5C$
T4	V_{AT4}	$V_{BT4}+C$

where V_{ATi} and V_{BTi} are T_i's V_A and V_B (i=1, 2, 3, and 4).

Proof:

We need to prove that no bidder of any type has an incentive to depart from the BNE. First we examine T1 bidders. Each T1 bidder's two bids equal their corresponding value plus 0.5C. For bidder *i* of T1 (1≤i≤N), the expected profit is

$$\pi_{T1i} = 0.75^{N-1} \cdot (V_{AT1} + V_{BT1} + C - b_A^{2nd} - b_B^{2nd})$$

Where b_A^{2nd} is the expected second highest b_A when bidder i (T1) is the only T1 bidder, b_B^{2nd} is the expected second highest b_B when bidder i (T1) is the only T1 bidder.

Please note that a T1 bidder has a profit only when all other bidders are not of T1. If there are more than one T1 bidders, say j, T1 bidders (2≤j≤N), each of them has 1/j chance of winning both products with an expected profit of zero and 1/(1+j) chance of losing both products with an expected profit of zero .

A T1 bidder has no incentive to move from this equilibrium, because increasing one or both her bids won't increase her expected profit. This is because when all her opponents are not of T1, it doesn't increase her profit. When there are other T1 bidders, increasing one or both her bids lets her win the bundle, but her profit is always zero since the price she pays always equals the value of the bundle Decreasing one or both her bids will not increase expected profit either. Because, again, when all her opponents are not of T1, it does not increase her profit when there are other T1 bidders. Decreasing one or both her bids lets her lose the bundle but her profit is always zero.

Likewise, we can apply the same logic to prove other types of bidder will not depart from the BNE. For a T2 bidder, the sum of b_A and b_B is equal to the value of the bundle. Only when all of her opponents are of T3 and/or T4, can a T2 bidder have a profit. Increasing her b_B will not increase her expected profit. Because when some of her opponents are of T1 and/or T2, she still either does not win any product or wins the bundle and pays a price equal to the value of the bundle. When her opponents are of T3 and/or T4, increasing b_B does not increase profits. Decreasing her b_A will not increase her expected profit, as she will lose A to T3 bidders and therefore lose C. When her opponents are of T1, T2 and T4, the results are not different. For the same reason, T4 bidders will not depart from the BNE.

For a T3 bidder, the sum of b_A and b_B is also equal to the value of the bundle. Only when all of her opponents are of T3, does she have a chance to win the bundle with zero profit. Increasing or decreasing one or both her bids does not increase her profit.

Q.E.D

Theorem 2 (Equilibrium Statement): *for two separate auction of N bidders, in scenarios 2 and 4, a BNE will consist of n type-contingent strategies.*

Type	b_A	b_B
T1	1	1, i = 1, 2,…, Nh* 1+2L+C, i = Nh+1,.., N
T2	C	1
T3	Bidder 1: -2L or C** Other bidders: C	C
T4	1	C

where V_{ATi} and V_{BTi} are T_i's V_A and V_B (i=1, 2, 3, and 4).

*Nh is the number of T1 bidders who bid 1 on B in BNE. Nh is given in the proof.
** b_A= -2L when $(1-1/N)(L+C)\geq(2^{(N-1)}-1)(-2L-C)$; b_A=C when $(1-1/N)(L+C)\geq(2^{(N-1)}-1)(-2L-C)$

Proof: For T2 bidders, the optimal strategy is to put all C on their bids for A, i.e. to bid (C, 1), to minimize the exposure risk. Similarly, for T4 bidders the optimal bids is to put all C on their bids for B, i.e. to bid (C, 1), to minimize the exposure risk.

For T3 bidders, there are two optional strategies that are not weakly dominated, (-2L, C) and (C, C). In a BNE, only one T3 bidder (say bidder 1) will choose one of these two bids, while all other N-1 bidders will always bid (C,C).

For bidder 1, the profit of bidding (-2L, C) is:

$$\pi_1 = 0.25^{N-1}\cdot(L+C)-(0.5^{N-1}-0.25^{N-1})\cdot(-2L-C),$$

where W1=L+C, L1=-2L-C;

The profit of bidding (C, C) is: $\pi_2 = 0.25^{N-1}\cdot\dfrac{(L+C)}{N}$, Only when $\pi 1 \geq \pi 2$, i.e. $(1-\frac{1}{N})(L+C) \geq (2^{N-1}-1)(-2L-C)$, bidder 1 of T3 will bid (-2L, C). All other T3 bidders would not increase their bids, since if they do so when bidder 1 is of T1, T3 and T4, they make zero profits, and if bidder 1 is of T2, they may lead a loss.

For T1 bidders, there are two optional strategies that are not weakly dominated, (1, 1+2L+ C) and (1, 1). In a BNE, depending on C and L, some of T1 bidders, say Nh of them, will choose the first strategy while all other (N-Nh) T1 bidders choose the second strategy.

(1) When $(1-\frac{1}{N})(L+C) < (2^{N-1}-1)(-2L-C)$ (so all T3 bidders bid (C, C)).

For the Nh T1 bidders, who choose to bid (1, 1), their profit is:

$$\begin{aligned}\pi_h^{Nh} &= (0.5^{N-1} - 0.25^{N-1})\cdot(1+2L)\cdot 2 + 0.25^{N-1}\cdot[2+4L)] \\ &\quad -(0.75^{N-1} - 2\cdot 0.5^{N-1} + 0.25^{N-1})\cdot(-2L-C) \\ &\quad -\sum_{m=1}^{Nh-1}\frac{Nh!}{(Nh-m)!m!}\cdot\frac{1}{1+m}\cdot 0.25^{N-1}(-2L-C)\end{aligned} \qquad (1)$$

Since

$$\sum_{m=1}^{Nh-1}\frac{Nh!}{(Nh-m)!m!}\cdot\frac{1}{1+m} = \sum_{m=1}^{Nh-1}\frac{Nh!}{(Nh-m)!(m+1)!}$$

Let m1=m+1, Nh1= Nh + 1,

$$\sum_{m=1}^{Nh-1}\frac{Nh!}{(Nh-m)!(m+1)!} = \sum_{m1=2}^{Nh}\frac{Nh!}{(Nh-m1+1)!(m1)!} = \sum_{m1=2}^{Nh1-1}\frac{Nh1!}{(Nh1-m1)!(m1)!}\cdot\frac{1}{Nh1}$$

Since

$$\sum_{m1=0}^{Nh1}\frac{Nh1!}{(Nh1-m1)!(m1)!} = 2^{Nh1}$$

$$\sum_{ml=2}^{Nhl-1} \frac{Nhl!}{(Nhl-ml)!(ml)!}\frac{1}{Nhl} = \frac{1}{Nhl}\left(2^{Nhl}-1-2Nhl\right) = \frac{1}{Nh+1}\left(2^{(Nh+1)}-2Nh-3\right)$$

So (1) becomes:

$$\begin{aligned}\pi_h^{Nh} &= (0.5^{N-1}-0.25^{N-1})\cdot(1+2L)\cdot 2+0.25^{N-1}\cdot[2+4L]\\ &-(0.75^{N-1}-2\cdot 0.5^{N-1}+0.25^{N-1})\cdot(-2L-C)\\ &-\frac{1}{Nh+1}\left(2^{(Nh+1)}-2Nh-3\right)\cdot 0.25^{N-1}\cdot(-2L-C)\end{aligned} \tag{2}$$

For the remainder of the (N-Nh) T1 bidders who choose to bid (1, 1+2L+ C), their profit is:

$$\pi_L^{Nh} = (0.5^{N-1}-0.25^{N-1})\cdot(2+3L-C)+0.25^{N-1}\cdot(2+4L) \tag{3}$$

At a BNE, no T1 bidders can increase her profit by switching strategies. It means $\pi_h^{Nh} \geq \pi_L^{Nh+1}, \pi_L^{Nh} \geq \pi_H^{Nh+1}$, i.e

$$\begin{aligned}&(2^{N-1}-1)\cdot(L+C)-(3^{N-1}-2\cdot 2^{N-1}+1)\cdot(-2L-C)\\ &-\frac{1}{Nh+1}\left(2^{(Nh+1)}-2Nh-3\right)\cdot(-2L-C)\geq 0\end{aligned} \tag{4}$$

$$(2^{N-1}-1)\cdot(L+C)-(3^{N-1}-2\cdot 2^{N-1}+1)\cdot(-2L-C)-\frac{1}{Nh+2}\left(2^{(Nh+2)}-2Nh-5\right)\cdot(-2L-C)\leq 0 \tag{5}$$

Using equations (4) and (5), we can calculate the BNE for Nh given L and C.

(2) When $(1-\frac{1}{N})(L+C) \geq (2^{N-1}-1)(-2L-C)$, one T3 bidder will bid higher than other T3 bidders. For T1 bidders, this change equally affects the profit of the two alternative strategies, so the T1 bidders' type contingent bids are the same as in (1).

Q.E.D

Theorem 3 (Equilibrium Statement): *For two separate auctions of N bidders, in scenarios 6 and 8, a BNE will consist of n type-contingent strategies.*

Types .	b_A	b_B
T1	V_{AT1}+0.5C	V_{BT1}+0.5C
T2,T4	0.5+0.5C	0.5+0.5C
T3	V_{AT3}++0.5C	V_{AT3}+0.5C

where V_{ATi} and V_{BTi} are T_i's V_A and V_B (i=1, 2, 3, and 4).

Proof: The reason that none of T1 and T3 bidders will depart from the BNE is essentially the same as that for T1 and T3 bidders in theorem 1.

A T2/T4 bidder has profit only when all other bidders are of T3. Because if there is more than one T2/T4 bidder, say j bidders (2≤j≤N), each of them has 1/j chance of winning both products with an expected profit of zero and 1/(1+j) chance of losing both products with an expected profit of zero .

A T2/T4 bidder has no incentive to move from this equilibrium because increasing one or both of her bids will not increase her expected profit. This is because when all her opponents are not of T2/T4, it doesn't increase her profit; when there are other T2/T4 bidders, increasing one or both her bids lets her win the bundle but her profit is always zero since the price she pays always equals the value of the bundle. Decreasing one or both her bids will not increase her expected profit either. Because again when all her opponents are not of T2/T4, it doesn't increase her profit, and when there are other T2/T4 bidders, decreasing one or both her bids lets her lose the bundle and her profit will always be zero.

Q.E.D

Appendix 2.5: Revenue Calculation When C>0

2.5.1 Revenue Calculation in Two Separate Auctions

2.5.1.1 Scenario 1, 3, 5 and 7

Based on Appendix 2.4 we know the BNE strategies of all four types and therefore all potential revenues. Figure 2.15 demonstrates all these potential revenues (the small triangles are potential revenues and small circles are type contingent strategies). Table 2.12(A) shows the amounts of all the potential revenues in Scenario 1, 3, 5 and Table 2.12(B) shows the amounts of all the potential revenues in Scenario 7. The calculation of the likelihoods of all potential revenues is exactly the same as when C=0 and L>0 (See Appendix 2.1, Table 2.5(A) and (B)). The expected revenue, given N and L, is equal to the weighted sum of all potential revenues.

Figure 2.15 Potential Revenues in Two Separate Auctions: Scenario 1, 3, 5 and 7

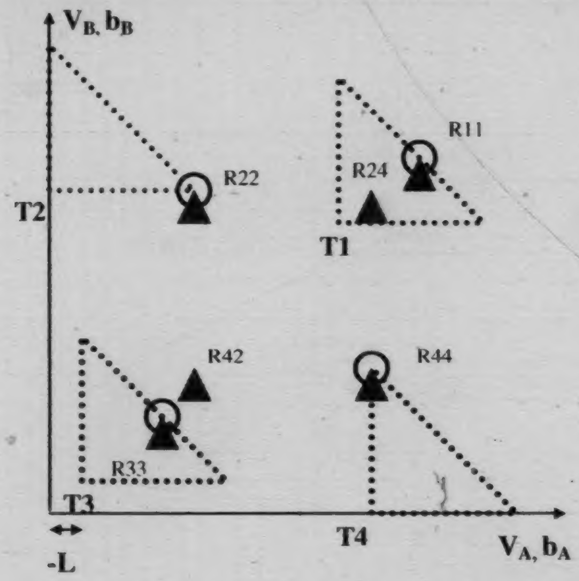


Table 2.12(A) Potential Revenues in Two Separate Auctions: Scenario 1, 3 and 5

Revenues	Amount
R33	-2L+C
R42	2L+2C
R22,R44	2-2L
R24	2
R11	2+C

NOTE: $L \leq 0$

Table 2.12(B) Potential Revenues of Two Separate Auctions: Scenario 7

Revenues	Amount
R33	C
R42	2L+2C
R22,R44	1+C
R24	2-2L
R11	2+C

NOTE: $L > 0$

2.5.1.2 Scenario 2 and 4

When $(1-1/N)(L+C) < (2^{N-1}-1)(-2L-C)$, all T3 bidders bid (C, C). Figure 2.16(A) demonstrates all these potential revenues (the small triangles are potential revenues and small circles are type contingent strategies). Table 2.13(A) and 2.13(B) show The calculation of the likelihoods of all potential revenues when N=2 and N>2. Table 2.13(C) shows the amounts of all the potential revenues in Scenario 2 and 4. For given N and L, the expected revenue is equal to the weighted sum of all potential revenues.

Figure 2.16(A) Potential Revenues in Two Separate Auctions: Scenario 2 and 4 when $(1-1/N)(L+C) < (2^{N-1}-1)(-2L-C)$

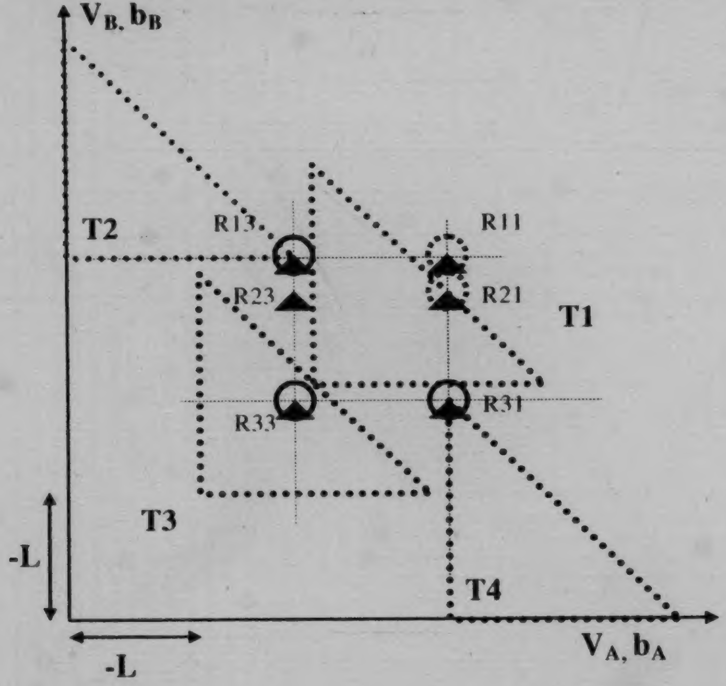


Table 2.13(A) Potential Revenues of Two Separate Auction and the Likelihoods: Scenario 2 and 4 when $(1-1/N)(L+C) < (2^{N-1}-1)(-2L-C)$ and N=2

Revenue	Likelihood	Explanation
R33	$2\cdot0.25\cdot0.25$ $+2\cdot0.25\cdot0.75$ $+0.25\cdot0.25$	One (T2), One (T4); Or One (T1), One (T2, T3, T4); Or Two (T1)
R23	0	N/A
R13	$2\cdot0.25\cdot0.25+0.25\cdot0.25$	One (T2), One (T1); Or Two (T2)
R31	$2\cdot0.25\cdot0.25+0.25\cdot0.25$	One (T4), One (T1); Or Two (T4)

Table 2.13(A) Potential Revenues of Two Separate Auction and the Likelihoods: Scenario 2 and 4 when $(1-1/N)(L+C)<(2^{N-1}-1)(-2L-C)$ and N=2(Continued)

R21	0	N/A
R11	$0.25\cdot0.25$	Two (T1)

Table 2.13(B) Potential Revenues of Two Separate Auction and the Likelihoods: Scenario 2 and 4 when $(1-1/N)(L+C)<(2^{N-1}-1)(-2L-C)$ and N>2

Revenue	Likelihood	Explanation
R33	$N\cdot0.25\cdot(N-1)\cdot0.25\cdot0.25^{(N-2)}$ $+N\cdot0.75\cdot0.25^{(N-1)}$ $+0.25^{N}$	One (T2). One (T4). Rest (T3): Or One (T1.T2.T4). Rest (T3): Or All (T3)
R23	$NL\cdot0.25\cdot(N-1)\cdot0.25\cdot0.25^{(N-2)}$	One L(T1). One(T2). Rest (T3)*
R13	$NH\cdot0.25\cdot(N-1)\cdot0.25\cdot0.25^{(N-2)}$ $+N\cdot0.5\cdot(0.5^{(N-1)}-0.25^{(N-1)}-(N-1)\cdot0.25\cdot0.25^{(N-2)})$ $+(0.5^{N}-0.25^{N}-N\cdot0.25\cdot0.25^{(N-1)})$	One (T2). One H(T1). Rest (T3):** Or Two (T2). One (T1. T4). Rest (T2.T3): Or Two (T2). Rest (T2.T3):
R31	$N\cdot0.25\cdot(N-1)\cdot0.25\cdot0.25^{(N-2)}$ $+N\cdot0.5\cdot(0.5^{(N-1)}-0.25^{(N-1)}-(N-1)\cdot0.25\cdot0.25^{(N-2)})$ $+(0.5^{N}-0.25^{N}-N\cdot0.25\cdot0.25^{(N-1)})$	One (T4). One (T1). Rest (T3) Or Two (T4). One (T1. T2), Rest (T3.T4): Or Two (T4). Rest (T3.T4)
R21	$NL\cdot0.25\cdot NH\cdot0.25\cdot0.25^{(N-2)}$ $+NL\cdot0.25\cdot(N-1)0.25\cdot[0.5^{(N-2)}-0.25^{(N-2)}]$ $+[0.75^{NL}-NL\cdot0.25\cdot0.5^{(NL-1)}-0.5^{NL}]\cdot0.5^{NH}$ $+[0.75^{NL}-NL\cdot0.25\cdot0.5^{(NL-1)}-0.5^{NL}]\cdot NH\cdot0.5\cdot0.5^{(NH-1)}$ $+NL\cdot0.25\cdot[0.75^{(NL-1)}-(NL-1)\cdot0.25\cdot0.5^{(NL-2)}-0.5^{(NL-1)}]\cdot0.5^{NH}$	One L(T1). One H (T1). Rest (T3.T4): Or One L(T1). One (T2).One (T4). Rest (T3.T4): Or Two L(T1). Rest L(T1. T3. T4). All H (T3. T4): Or Two L(T1). One H(T1. T2). Rest L(T1. T3.T4).Rest H(T3.T4): Or Two L(T1). One L(T2). Rest L(T1, T3,T4). ALL H(T3.T4)
R11	$[1-NH\cdot0.25\cdot0.75^{(NH-1)}-0.75^{NH}]$ $+NH\cdot0.25\cdot[0.75^{(NH-1)}\cdot1-0.75^{NL}\cdot0.75^{(NH-1)}-0.75^{NL}\cdot0.5^{NH-1}+0.5^{(NH-1)}\cdot0.5^{NL}]$ $+NH\cdot0.25\cdot[0.75^{(N-1)}\cdot1-0.5^{(N-1)}-0.5^{(N-1)}+0.25^{N-1}]$ $+[0.75^{NH}-0.5^{NH}\cdot0.75^{NL}-(NH\cdot0.25\cdot0.5^{(NH-1)}\cdot0.75^{NL}-NL\cdot0.25\cdot0.75^{(NL-1)}\cdot0.5^{NH})-0.75^{NH}\cdot0.75^{NL}-NL\cdot0.25\cdot0.75^{NH}\cdot0.75^{(NL-1)}+0.5^{NH}\cdot0.5^{NL}+(NH\cdot0.25\cdot0.5^{(NH-1)}\cdot0.5^{NL}-NL\cdot0.25\cdot0.5^{(NL-1)}\cdot0.5^{NH})+NL\cdot0.25\cdot0.5^{NH}\cdot0.5^{(NL-1)}]$ $+NL\cdot0.25\cdot[0.75^{N-1}-0.5^{N-1}-0.5^{N-1}-(N-1)\cdot0.25\cdot0.5^{(N-2)}+0.25^{N-1}+(N-1)\cdot0.25\cdot0.25^{(N-2)}+(N-1)\cdot0.25\cdot0.25^{(N-2)}+(N-1)\cdot0.25\cdot(N-2)\cdot0.25\cdot0.25^{(N-3)}]$ $+[0.75^{N}-0.5^{N}-0.5^{N}-N\cdot0.25\cdot0.5^{(N-1)}-N\cdot0.25\cdot0.5^{(N-1)}+0.25^{N}+N\cdot0.25\cdot0.25^{(N-1)}+N\cdot0.25\cdot0.25^{(N-1)}+N\cdot0.25\cdot(N-1)\cdot0.25\cdot0.25^{(N-2)}]$	Two H(T1), Rest (ALL); Or One H (T1). One L (T1). One (T2). Rest L(all). Rest H(T2, T3. T4): Or One H (T1). One (T4). One (T2). Rest (T2. T3. T4): Or Two L(T1). Two(T2), Rest L(ALL). Rest H (T2.T3.T4): Or One L(T1). Two(T2). One (T4). Rest (T2.T3.T4): Or Two(T2). Two (T4). Rest (T2.T3.T4)

Note: In the BNE, for bidders of T1, NH of N bidders will bid (1, 1) and NL bidders will bid (1, 1+2L+C).

* One L (T1) means one T1 bidder bids (1, 1+2L+C)

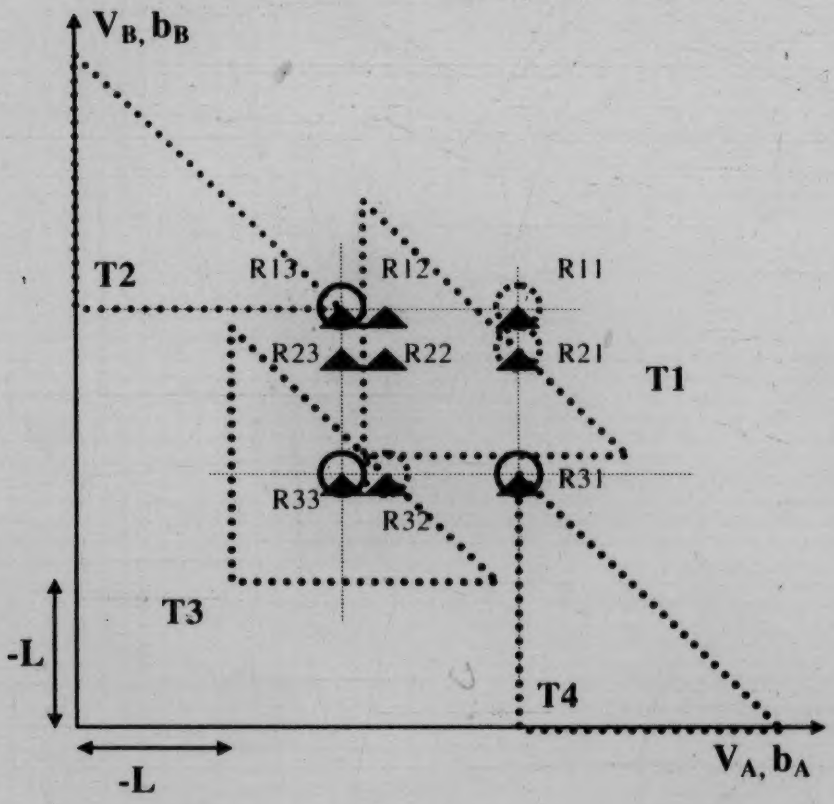
** One H (T1) means one T1 bidder bids (1, 1)

Table 2.13(C) Potential Revenues of Two Separate Auctions: Scenario 2 and 4 when $(1-1/N)(L+C) < (2^{N-1}-1)(-2L-C)$

Revenues	Amount
R33	2C
R23	1+2L+2C
R13,R31	1+C
R21	2+2L+C
R11	2

When $(1-1/N)(L+C) \geq (2^{N-1}-1)(-2L-C)$, one of the T3 bidders (say bidder 1) will bid (-2L, C) and the rest of the T3 bidders will bid (C, C). Figure 2.16(B) demonstrates all these potential revenues. Table 2.13(D) shows the amounts of all the potential revenues for Scenarios 2 and 4. The expected revenue, given N and L, is equal to the weighted sum of all potential revenues. The only difference in the revenue is caused by the change in bidder 1's bids when she is of T3 (1/4 chance). Therefore the calculation of expected revenue is similar as the previous case. We just need to replace R13 by (3/4·R13+1/4·R12), replace R23 by (3/4·R23+1/4·R22), and replace R33 by (3/4·R33+1/4·R32) in table 2(C) to reflex the change in bidder1's strategy. We can take advantage of Table 2.5(A) and 2.5(B) to get the expected revenue when $(1-1/N)(L+C) \geq (2^{N-1}-1)(-2L-C)$.

Figure 2.16(B) Potential Revenues in Two Separate Auctions: Scenario 2 and 4 when $(1 - 1/N)(L + C) \geq (2^{N-1} - 1)(-2L - C)$



V_B, b_B
T2
R13
R12
R11
R23
R22
R21
T1
R33
R32
R31
T3
-L
T4
-L
V_A, b_A

Table 2.13(D) Potential Revenues of Two Separate Auctions: Scenario 2 and 4 when $(1 - 1/N)(L + C) \geq (2^{N-1} - 1)(-2L - C)$

Revenues	Amount
R33	2C
R32	-2L+C
R23	1+2L+2C
R13,R31	1+C
R22	1+C

Table 2.13(D) Potential Revenues of Two Separate Auctions: Scenario 2 and 4 when $(1-1/N)(L+C) \geq (2^{N-1}-1)(-2L-C)$ (Continued)

R21	2+2L+C
R12	1-2L
R11	2

2.5.1.3 Scenario 6 and 8

Figure 2.17 demonstrates all the potential revenues for scenarios 6 and 8. Table 2.14(A) and (B) show the calculation of the likelihoods of all potential revenues when N=2 and N >2. Table 2.14(C) shows the amounts of all the potential revenues for Scenario 6, and Table 2.14(D) shows the amounts of all the potential revenues for Scenario 8. The expected revenue, given N and L, is equal to the weighted sum of all potential revenues.

Figure 2.17 Potential Revenues in Two Separate Auctions: Scenario 6 and 8

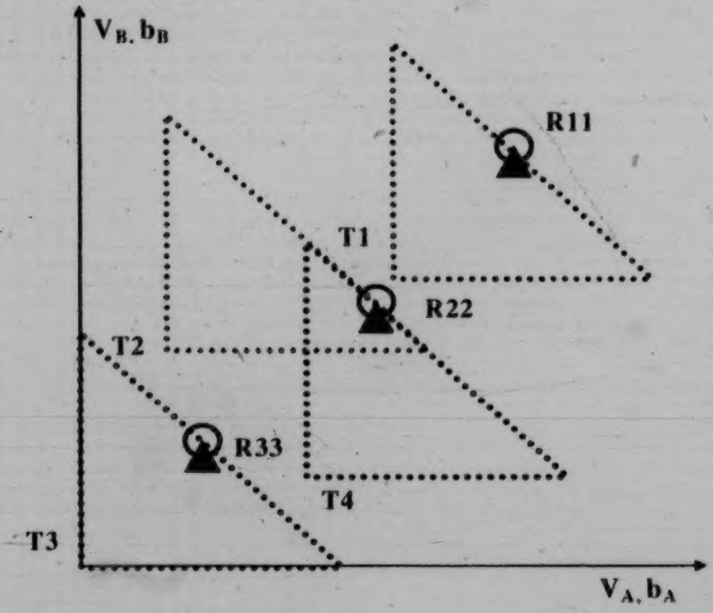


Table 2.14(A) Potential Revenues of One Bundled Auction and the Likelihoods: Scenario 6 and 8 When N=2

Revenue	Likelihood	Explanation
R11	$0.25 \cdot 0.25$	Two (T1)
R22	$2 \cdot 0.5 \cdot 0.25$ $+0.5^2$	One (T2,T4), One (T1); Or Two (T2,T4)
R33	$2 \cdot 0.75 \cdot 0.25$ $+0.25^2$	One (T1,T2,T4), One (T3); Or Two (T3)

Table 2.14(B) Potential Revenues of One Bundled Auction and the Likelihoods: Scenario 6 and 8 When N>2

Revenue	Likelihood	Explanation
R11	$1-0.75^N-N \cdot 0.25 \cdot 0.75^{(N-1)}$	Two (T1), Rest (ALL)
R22	$N \cdot 0.25 \cdot [0.75^{(N-1)}-0.25^{(N-1)}]$ $+[0.75^N-0.25^N-N \cdot 0.5 \cdot 0.25^{(N-1)}]$	One (T2,T4), One (T1), Rest (T2,T3,T4); Or Two (T2,T4), Rest (T2,T3,T4)
R33	$N \cdot 0.75 \cdot 0.25^{(N-1)}$ $+0.25^N$	One (T1,T2,T4), Rest (T3); Or All (T3)

Table 2.14(C) Potential Revenues of Two Separate Auctions: Scenario 6

Revenue	Value
R11	2+2L+C
R22	1+C
R33	-2L+C

Note: $L \leq 0$

Table 2.14(D) Potential Revenues of Two Separate Auctions: Scenario 8

Revenue	Value
R11	2+C
R22	1+C
R33	C

Note: L>0

2.5.2 Revenue Calculation in One bundled auction

Figure 2.18 demonstrates all the potential revenues for a bundled auction. Table 2.15(A) shows the amounts of all the potential revenues when -0.5≤L≤0, and Table 2.15(B) shows all the potential revenues when 0.5≥L>0. Tables 2.15(A) and (B) show the calculation of the likelihoods of all potential revenues when N=2 and N >2. Table 2.15(C) shows the amounts of all the potential revenues when complementarity is present. Again, expected revenues are equal to the weighted sum of all potential revenues.

Figure 2.18(A) Revenues When -0.5≤L≤0

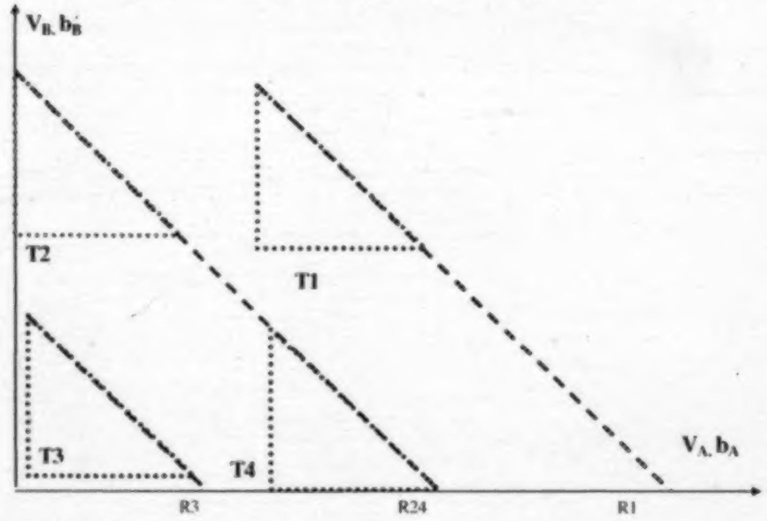


Figure 2.18(B) Revenues When 0.5≥L>0

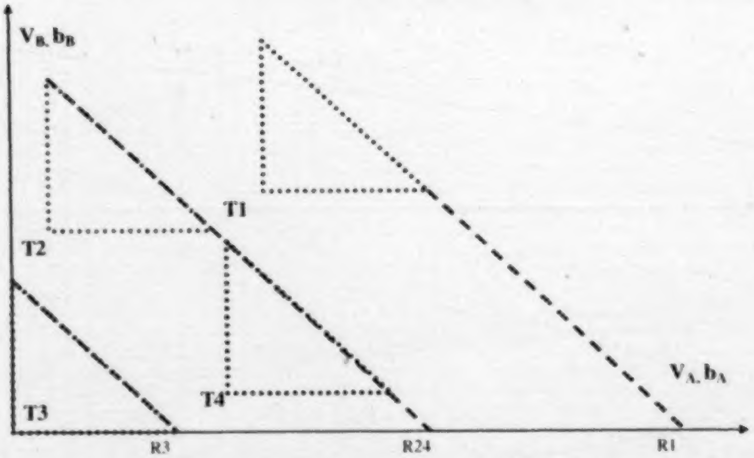


Table 2.15(A) Potential Revenues of One Bundled Auction and the Likelihoods When C=0 and N=2

Revenue	Likelihood	Explanation
R1	$0.25\cdot0.25$	Two (T1)
R24	$2\cdot0.5\cdot0.25$ $+0.5^2$	One (T2,T4), One (T1); Or Two (T2,T4)
R3	$2\cdot0.75\cdot0.25$ $+0.25^2$	One (T1,T2,T4), One (T3); Or Two (T3)

Table 2.15(B) Potential Revenues of One Bundled Auction and the Likelihoods When C=0 and N>2

Revenue	Likelihood	Explanation
R1	$1-0.75^{N}-N\cdot0.25\cdot0.75^{(N-1)}$	Two (T1). Rest (ALL)
R24	$N\cdot0.25\cdot[0.75^{(N-1)}-0.25^{(N-1)}]$ $+[0.75^{N}-0.25^{N}-N\cdot0.5\cdot0.25^{(N-1)}]$	One (T2,T4), One (T1), Rest (T2.T3.T4): Or Two (T2,T4). Rest (T2,T3,T4)
R3	$N\cdot0.75\cdot0.25^{(N-1)}$ $+0.25^{N}$	One (T1,T2.T4). Rest (T3); Or All (T3)

Table 2.15(C) Potential Revenues of One Bundled Auction When C>0

Revenue	Amount (L>0)	Amount (L≤0)
R1	2+C	2+2L+C
R24	1+C	1+C
R3	C	-2L+C

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Chapter 3

Essay 2: Why More Can Be Less: An Inference-Based Explanation for Hyper-Subadditivity in Bundle Valuation

3.1 Introduction

People should not prefer less to more. They certainly should not be willing to pay more for less. Despite this, work in several disciplines has demonstrated that under certain circumstances, people prefer less to more, or are willing to pay more for less (see Hsee 1998; Simonson, Carmon and O'Curry 1994; Johnson, Hershey, Meszaros, and Kunreuther 1993). While such findings have received both empirical and theoretical attention, the more-is-less phenomenon has usually been considered as an antecedent state to some other phenomenon of primary interest (e.g. "preference reversal"). As such, the more-is-less phenomenon has not been the central focus of most previous investigation, and little if any theoretical or empirical work has been done to investigate the underlying processes which might lead people to prefer less to more.

Our focus, therefore, is not on the resultant preference reversal, but rather on the antecedent phenomenon itself. We wish to determine how an entity with positive value on its own can have negative marginal value in a bundle. Precisely how can more be less? We refer to this previously unexplained phenomenon as "hyper-subadditivity" to facilitate comparisons with better-known concepts such as additivity, subadditivity and superadditivity. In order to fully test the framework we develop, we also consider the flip side of the "more-is-less" effect, superadditivity without complementarity. This occurs when the sum of the components combined (the bundle) is greater than the sum of the individual components added together, even though the items are not complementary. As it turns out, a value inferencing model can explain both of these situations, whereas basic economic theory cannot.

The paper proceeds as follows. We review the literature relating to: preference reversals, other more-is-less phenomena, departures from the economically rational expectation of additivity, and the extant bundling literature. We then introduce concepts from the inference-making literature, and explain how inference making can lead to hyper-subadditivity, and superadditivity without complementarity. We then test this inference-based model of bundle assessment in a series of three controlled field experiments involving auctions. We conclude the paper with the implications of these studies and recommendations for future research.

3.2 Literature Review

3.2.1 Preference Reversals

For years researchers have demonstrated that decision makers often depart in practice from the rigid expectations of economic theory. "Preference reversals" are one such well-known departure. Preference reversals refer to a situation where A is preferred to B under one assessment method, but B is preferred to A under another assessment method (see Irwin 1994; and Hsee, Blount, Loewenstein, and Bazerman 1999 for a review). Within this literature, an additional departure from the expectations of economic theory has been observed. This departure has been termed the "more-is-less" effect (List 2002), or, alternatively, as the "less is better" effect (Hsee 1998; Simonson, Carmon and

O'Curry 1994; Johnson et al. 1993). When assessing alternatives independently, people actually prefer less to more. That is, something with positive utility on its own is perceived to have negative marginal utility when added to a bundle. People prefer less to more when assessing the two bundles independently, but prefer more to less when assessing the same bundles concurrently.

For example, Hsee (1996, and 1998) and Hsee et al. (1999) demonstrated that a normatively less valuable item can be preferred to a more valuable option, when options were presented separately. Specifically, Hsee showed that respondents preferred a complete 24-piece dinner set over a set with 31 intact pieces and several broken ones (which included the same 24 intact pieces). Likewise, an over-filled 7-ounce serving of ice cream was preferred to an under-filled 8-ounce serving. He proposed that these results are based on the evaluability hypothesis, which argues that separate evaluations of items tend to be influenced by easy-to-evaluate attributes rather than salient ones. Hsee found that when the two sets are presented side by side, most people prefer more to less. Both the broken-dish set and the under-filled cone, however, clearly establish a reference point against which the actual offering performs poorly. If, however, decision makers use the value of certain items to infer the value of uncertain items, this requirement becomes unnecessary. To test this in our studies, we use super-ordinate sets that do not establish such a reference point.

Until now, the bulk of theoretical discussion has centered on the preference reversal between independent and concurrent assessment, rather than on the more-is-less phenomenon itself. More-is-less theory has become an integral part of the preference reversal literature. While several theories have been proposed to explain the preference reversals associated with the more-is-less effect (for a review of the preference reversal literature see Hsee et al. 1999), little attention has been given the question of why more should ever be less in the first place.

3.2.2 Other More-is-Less Findings

In addition to the preference reversal literature, several published studies have identified other situations in which more is less. These examples come from the psychology, decision making and marketing literatures. These studies can be divided into two general classes: unpacking and dilution.

The unpacking principle (associated with support theory) can lead to the more-is-less phenomenon. This principle suggests that unpacking the description of a risk into specific parts can increase judged probabilities (Tversky and Koehler 1994; Rottenstreich and Tversky 1997). This may lead to violations of monoticity in judgments of probability as well as the pricing of uncertain prospects. Johnson et al. (1993) provide a good example of how the unpacking principle can lead to hyper-subadditivity. They observed that participants were willing to pay more for health insurance covering "any disease or accident" than a policy covering "any reason." While unpacking can certainly lead people to pay more for less, it seems that this type of hyper-subadditivity is most likely to apply in situations, such as insurance or extended warranties, where the cueing provided by the unpacked subordinate set makes people think about all the factors that could go wrong. Our investigation of hyper-subadditivity, therefore, carefully avoids situations where unpacking might provide an alternative explanation for the results.

Dilution also allows adding something of non-negative value to reduce bundle attractiveness. Simonson et al. (1994) studied the effect of adding an unneeded feature or sales promotion to the choice of an existing item. They concluded that additional unneeded features or promotions, which are free or optional, can decrease the choice probability of the enhanced item. It is not clear, however, whether this phenomenon requires that the added feature or component be "unneeded." Simonson et al. (1994), in fact, highlight the need for future research to replicate their results with real item purchases. Having discussed extant more-is-less results, we turn now to the economic concepts of additivity, subadditivity, and superadditivity, so as to place hyper-subadditivity within this well established framework.

3.2.3 Departures from Additivity

Additivity is the default economic assumption for the value of a bundle of goods. It implies that the value of a bundle is equal to the sum of the values of the individual components (Adams and Yellen 1976; Schmalensee 1984). Two general departures from additivity have been described: superadditivity, where the value of the bundle is greater than the sum of its parts, and subadditivity, where the value of the bundle is less than the sum of its parts.

Economic theory can readily address superadditivity through the introduction of the concept of complementarity. If two or more items in a group (i.e., a bundle) are complements, then ownership of one item in the bundle enhances the utility of another item therein. A contemporary example of bundle complementarity leading to superadditivity is a digital camera and color printer, which share proprietary software. By purchasing both, a decision maker can gain benefits not available by mixing the camera of company A with the printer of company B. Thus, the value of the combined camera and printer may be higher than the sum of the individual values of each item. Examples of superadditivity due to the complementarity of goods can be found in Guiltinan (1987) as well as Harlam, Krishna, Lehman and Mela (1995).

Economic theory can also address the opposite: subadditivity. In this case, the value of the bundle is lower than the sum of the values of each component. The introduction of the concept of substitutability makes this possible. Bundle item substitutability is determined by the overlap in their sources of utility (i.e., benefit overlap). Because of this benefit overlap, the value of the bundle to an individual decision maker is not as high as the sum of her/his values for the individual items. An example of subadditivity due to substitutability might be a snowboard and a pair of skis. While a decision maker might have high utility for either item, the utility of the two combined might be much less than the sum of the utility of each one. Subadditivity of decision maker valuation due to the substitutability of goods has recently been demonstrated by Cooke and Pecheux (2001).

Hyper-subadditivity is one step beyond subadditivity. In subadditivity, the value of multiple objects is less than the sum of their individual values. In hyper-subadditivity, the value of multiple objects is less than the value of a subset of those objects. Thus, adding an item or items (with positive value) actually decreases the value of the bundle. More is less!

Economic concepts such as substitutability and complementarity cannot account for hyper-subadditivity. We believe that understanding the "more-is-less" phenomenon requires an explanation found not in the intrinsic qualities of the bundle, but in the processes employed by the person assessing that bundle.

3.2.4 Prior Bundling Research

Behavioral scientists have performed experiments to determine how decision makers evaluate various bundling formats (Gaeth, Levin, Chakraborty, and Levin 1990; Harlam et al. 1995; Johnson, Herrmann, and Bauer 1999; Yadav 1994). This stream of research has applied prospect theory (Kahneman and Tversky 1979), mental accounting (Thaler 1985) and anchoring and adjustment heuristics (Tversky and Kahneman 1974; Yadav 1994; Chapman and Johnson 1999) to explain how decision makers evaluate bundles of goods. Prospect theory proposes that because prices are perceived as losses, a bundle with a single price is more attractive than separate components with multiple prices (see, for example, Johnson et al. 1999).

Most of the previous bundling literature has focused on either complementarity or substitutability of items. It relies upon the relationship between the items to account for departures from additivity. The current research differs in that we rely upon inferential processes by the bundle assessor to show departures from additivity. Specifically, our model predicts that both superadditivity and hyper-subadditivity can be found when bundling items that are neither complements nor substitutes. Before describing this model, we briefly discuss previous work on inferences relevant to our model.

3.2.5 Decision Maker Inferences and Bundle Valuation

It is widely accepted that decision makers often have difficulty assessing the value of goods and services (e.g., Simonson and Tversky 1992), and that they frequently construct preferences when faced with making a decision (Tversky, Sattath, and Slovic 1988; Bettman, Luce, and Payne 1998). Previous research has shown that individuals infer missing or incomplete data from other information provided (see, for example, Anderson 1982; Johnson and Levin 1985;Levin, Johnson, Deldin, Carstens, Cressey, and Davis 1986; Wills and Moore 1996; Ebenbach and Moore 2000). Prelec, Wernerfelt and Zettelmeyer (1997) show that subjects who are unsure of item valuations will rely on the context, like the choice set, to make inferences about values. Not only do decision makers infer missing attributes from known attributes of a particular brand, but they can also assume missing attributes from information about other brands (Moon and Tikoo 1997).

In assessing a bundle of goods, decision makers may well use information from one of the bundled items to make inferences about other items in the bundle. Theoretically, this should be more likely when decision makers are more certain about the value of some goods over others (uneven value uncertainty). Because of this, we propose that decision makers use the value of high-certainty goods to make inferences about the value of less-certain goods (e.g., Johnson and Levin 1985; Simonson et al. 1994; Moon and Tikoo 1997). Uneven value uncertainty could lead to hyper-subadditivity if the inferred negative value of one item compared to a second item exceeds the positive value of the initial item.[15]

3.3 Conceptual Model and Hypotheses

The proposed model considers the question of how decision makers form valuations for multiple-item bundles under conditions of uneven value uncertainty (when items with low-value uncertainty are bundled with items with high-value uncertainty). We propose that under these conditions, the value of the more certain item may be an informative indicator of the value of the less certain item.

First we consider the situation where a low-value, low-uncertainty item is bundled with a high-value, high-uncertainty item. Figure 3.1 illustrates individual level value distributions for two items, where the width of the distribution indicates a decision maker's degree of uncertainty concerning the value. Item 1 has a low value and a low degree of uncertainty (a narrow value distribution), while item 2 has a high value and a high degree of uncertainty. In this case the low value of item 1 will be an informative indicator of low value, with the low level of uncertainty making it a strong indicator. Decision makers may then use this informative indicator to form their valuation of item 2, leading to a shift in the distribution for item 2 and a subsequent reduction in its value (see the dotted line in Figure 3.1).

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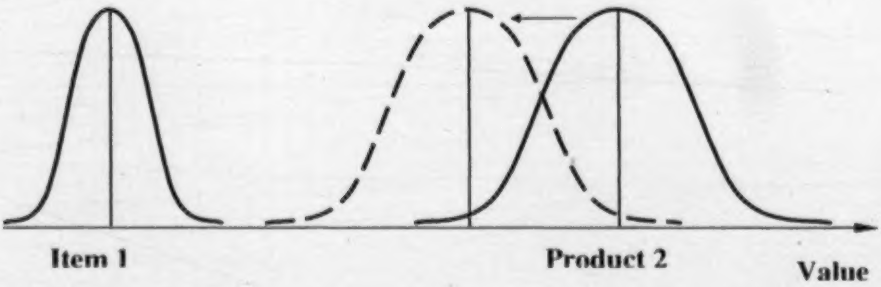
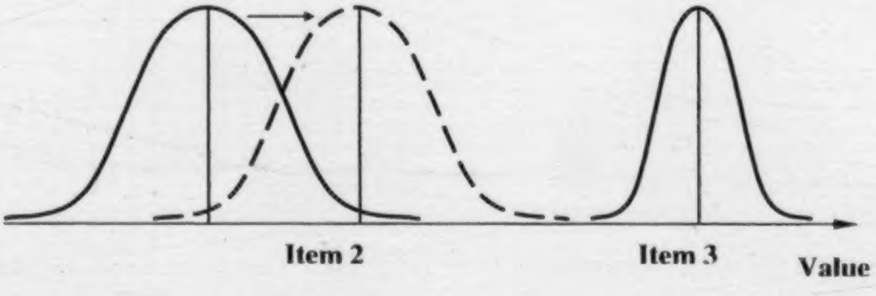
[15] Our interpretation is related to the decision maker inference explanation presented by Simonson et al. (1994) and evaluability theory presented by Hsee (1998). Simonson et al. (1994) argue that adding unneeded features or promotions may lead to inferences about the item's value or quality, and decision makers may use this as an indication of low quality. Evaluability theory explains a reversal in joint versus separate evaluations, depending on whether an attribute is easy or difficult to evaluate independently. Therefore, the value of an item, which is difficult to determine in isolation, can be influenced when bundled with a high-certainty item. However, neither interpretation explains hyper-subadditivity adequately.

Figure 3.1 Individual Level Value Distributions (items 1 and 2)



Next we consider the case where the certain item (item 3) is high-value in nature (Figure 3.2). Under this condition, the certain item constitutes a high-value indicator, shifting the value distribution for item 2 to the right. Thus, uneven value uncertainty can lead to the opposite outcome: superadditivity. In this way, our model permits superadditivity without complementarity. To provide the most conservative test possible of both implications of the uneven value uncertainty mechanism, we always use bundle components that are neither complements nor substitutes.

Figure 3.2 Individual Level Value Distributions (items 2 and 3)



In order to support this model of the decision maker bundle valuation process, our studies must demonstrate several points. To begin with, we should be able to determine that most decision makers arrive at their valuation of a bundle by first assessing the value of the individual components. Next, we must be able to show that under conditions of heterogeneity in item uncertainty (i.e., one item is more uncertain than the other), decision makers will focus on the more certain item in the bundle (e.g. consider it first), even if this is not the most expensive or important component. If the certain item has a low value, it should have a negative impact on the value of the uncertain item. In contrast, if the certain item has a high value, it should have a positive impact on the uncertain item. In the extreme case, this process might lead to hyper-subadditivity and superadditivity respectively.

Our conceptualization makes four important contributions to the literature. First, we demonstrate that in cases of high uncertainty about the value of an item in a bundle, another item can influence the value of the first item so adversely that its net impact on the bundle valuation is negative. Second, we show that bundling a high value high certain item with a low value low certain item leads to superadditivity, even though the items are non-complementary. Third, we employ psychological process measures which demonstrate that the majority of people begin by considering the most certain bundle item, even if it is less expensive or less important than the other items. Fourth, our results differ from those of Yadav (1994), who proposed and found that decision makers anchor on the item perceived to be most important (i.e., most expensive).

Next we discuss experiments designed to test these hypotheses. The objective of the first experiment is to test for the existence of hyper-subadditivity and to provide insights into how decision makers form valuations of a bundle of goods. Experiment 2 follows up by investigating this process with both high value and low value, high-certainty goods. Experiment 3 demonstrates how hyper-subadditivity and superadditivity of bundle valuations can be eliminated.

3.4 Experiments

3.4.1 Experiment 1

For the first experiment we conducted a series of second price, sealed-bid (Vickrey) auctions. In a Vickrey auction, the winner is the highest bidder, but s/he only pays the price equal to the second-highest bid (Vickrey 1961). We believe this method of obtaining customers' valuations (or willingness to pay) is highly desirable for several reasons. First, it has been shown that in Vickrey auctions it is always optimal for participants to bid their true value (Milgrom 2004).[16] Additionally, because these are real auctions where bidders are committing their own money, the bids represent economically-consequential assessments of value. Therefore, problems based on the inconsequentiality of certain methods are avoided (Jorgensen et al. 2004).

The bidders were 90 undergraduate business students at a major North American university. Participants were provided with detailed instructions, and were shown an example of a second-price, sealed-bid auction. They were told that if they were the highest bidder, they would be expected to pay for the item upon delivery (the following week) and that acceptable forms of payment were personal checks and cash (i.e. real auctions using real money). We explained to them that (1) there were several different auctions taking place, (2) they would each be randomly assigned to one or two of these auctions (depending on condition – bundled or unbundled), and (3) they would be competing with about 9 other bidders. They were also told that all items would be sold to the highest bidder regardless of price (i.e., there was no reserve price). Half of the participants were assigned to the unbundled condition, where they bid on the two items separately (giving a bid for each item). The other half of the participants were assigned to the bundled condition. The latter gave a single bid for the two-item bundle. Thus, the experiment employed a simple single-factor, two-level design. Participants were provided with a picture and a description of the item(s) on an 8.5 in. x 11 in. sheet of paper, and were asked to write their bid amount in the space provided at the bottom of the sheet. For participants in the unbundled condition, the order of the two auctions was randomized. After participants made their bid(s), they were asked to estimate the range of the local retail prices for the items they had bid on (i.e., the high and low retail price locally).

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[16] In addition, Palfrey (1983) has shown that it is also an optimum strategy for bidder to bid their value for a bundle of goods in second-price sealed bid auctions. While in empirical studies bidders may bid less than their value due to budget constraints, we control for this in the empirical analyses.

The two items used, were intentionally selected for their mean values and the uncertainty surrounding those values. In addition, we selected items that were neither substitutes (to avoid inherent subadditivity) nor complements (to avoid inherent superadditivity). Pre-testing was conducted to determine one low-value, low-uncertainty item (a spindle of 50 blank CDs) and one high-value, high-uncertainty item (a 16-piece knife set). The 50 CDs were described as lower quality units made in India, while the knives were described more ambiguously concerning value. Most students are familiar with 50-CD spindles, and were quite certain about their estimates of the local retail price. However, participants were far less certain about the value of the knife set. The average uncertainty of the CDs was significantly lower ($21.70) than was the average uncertainty about the knife set ($70.92).[17] This difference is statistically significant ($t = -5.24$, $df = 75$, $p < .01$). The standard deviation of the bid amount for the CDs was significantly lower than that of the knife set (6.99 versus 55.21).

Results

The results of Experiment 1, summarized in Figure 3.3, are based on 67 out of the 90 respondents who placed a bid in the auction.[18] Consistent with our expectations, there is evidence of hyper-subadditivity, as the average bid for the knife set was $38.62 while the average bid for the bundle with the knife set and CDs combined was $20.42. That is, people in the unbundled condition actually bid more for the knives than people in the bundled condition bid for both the knives and the CDs, and the difference is statistically significant (independent-samples one-tail t-test = -1.73, $df = 75$, $p < .05$). These people bid more for less.

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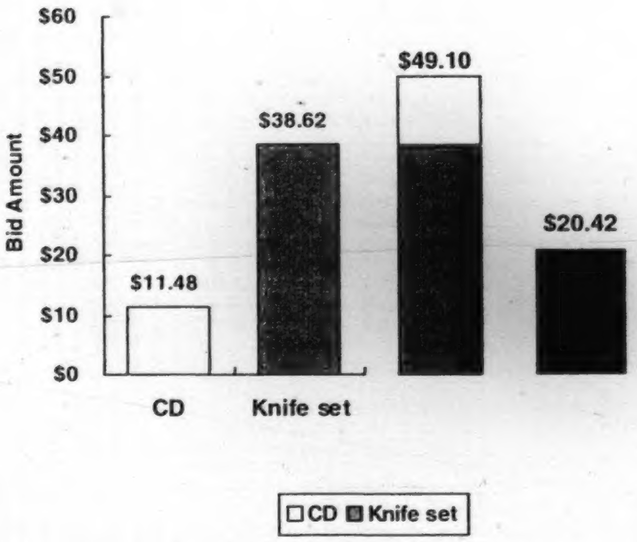
[17] From the post-hoc survey we obtained measures of the range of retail prices from each bidder. The average uncertainty is the summed differences of bidders' estimates of the high and the low in the retail prices.

[18] Thirteen participants opted not to place a bid in the auction, while another ten bidders placed insignificantly low bids, which was less than five percent of the highest bid. These bidders were about equally distributed over the two conditions, and their estimates of the minimum and maximum retail value did not differ considerably from those of the other bidders. Therefore, these deleted bids do not significantly influence the results.

Figure 3.3 Average Bid Amounts for Experiment 1



In addition to bidding, participants in the bundled condition also answered questions concerning the process/procedure used to determine their valuation of the bundle. We started with the following open-ended question: "How did you determine your estimate of the (combined) value of the CD-R Media 50-CD Spindle and the 16-piece cutlery set"? Of 40 completed observations, the most widely sited response was that participants first considered the CD spindle for which they knew the price, then estimated the value of the knife set, and finally combined both values (n = 21). Participants were also asked their level of agreement on a 10-point scale (from strongly disagree = 0, to strongly agree = 10) to the following statements: 1. I just came up with the overall value of the bundle (Mean = 2.62 and Standard Deviation = 2.40, and which is statistically different from the midpoint, t = 6.27, df = 39, P < 0.001). 2. I added my estimates for each of the individual items together (Mean = 7.41, Standard Deviation = 2.36, and which is statistically different from the midpoint, t = 6.46, df = 39, P < 0.00).

These results indicate that decision makers estimate the value of the individual items in the bundle and add these to form their value for the bundle of the goods, which is consistent with our conceptual model.

Discussion

Our first attempt to experimentally generate hyper-subadditivity based on value uncertainty was successful. The mean bid for the knife set alone was almost 90% higher than the average bid for the bundle of the knife and the CDs. We also demonstrate for the first time that people do in fact assess individual bundle components, and then combine these values to arrive at an overall bundle value. This had previously been merely an untested supposition. Given these initial encouraging results, in Experiment 2 we attempt a balanced replication (looking for both hyper-subadditivity and superadditivity) with the addition of measures that allow a deeper investigation of the underlying process.

3.4.2 Experiment 2

The objective of the second experiment is to further test our hypothesis that bidders infer value of low certain-value goods from high certain-value goods. To do this, we investigate the impact of both a positive value indicator and a negative value indicator on a single uncertain item. Specifically, we bundle the same uncertain item with either (1) an item lower in value but high in value certainty, or (2) an item higher in value and high in value certainty. We expect that if these two high-value certainty items provide a sufficiently strong indication about the low certainty item, then in the first case we may see hyper-subadditivity, while in the second we may in fact see superadditivity. Finding evidence of both would be a convincing demonstration of our conceptual model, but it is a rather conservative test.

In addition to looking for bi-directionality in the inference of value, we also wish to explore the mechanisms underlying any value inference phenomena observed. Specifically, we want to learn whether decision makers do in fact focus on the more certain item in the bundle, and whether they do so regardless of importance or price.

As in Experiment 1, we used an actual Vickrey (second price sealed-bid) auction to determine participants' valuations. In addition to the CDs and the knife set used in Experiment 1, participants in this experiment also bid on a high-value item with low value uncertainty (a Toshiba DVD player).[19] We used a mixed design, where each participant bid on all three items, but in one of three different bundle groupings. That is, each participant bid on one of the three items in isolation, as well as bidding on the other two in a bundle. Thus, all respondents participated in two auctions: one auction for a single item and a second auction for a bundle consisting of the two other items. The order of presentation was counterbalanced. At the completion of the auction, participants filled out a survey with questions about all the items in the auctions. They were also asked which item they focused on when bidding on the bundle.

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[19] Both the average bid amount and the average retail price indicate that the CDs are low value, the knife set medium value and the DVD player high value. Furthermore, we measured the degree of certainty about their estimate of the retail price (where 0 = very uncertain and 10 = very certain.). Participants were more certain about the value of the DVD (M = 5.74) and the CDs (M = 5.43), and less certain about the value of the knife set (M = 3.89). The differences between DVD and knife (t = -6.53, df =245, p < .01) and CDs and knife (t = 5.26, df = 244, p < .01) are statistically significant, while the difference between CDs and DVD is not significant (t= -.94, df = 247, p < .35).

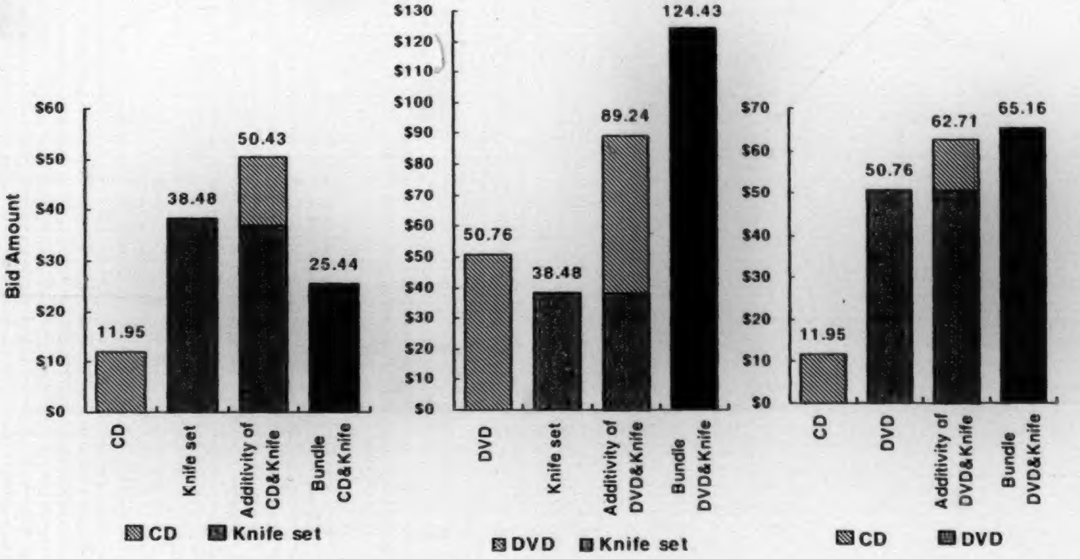
Results

One hundred and thirty-six undergraduate business students participated in this study, of which 121 placed meaningful bids in the auctions. The results of Experiment 2 are summarized in Figure 3.4. As in Experiment 1, the results are consistent with hyper-subadditivity when the low value (high certainty) CDs are bundled with the medium value (low certainty) knife set. The knife set sold at an average price of $38.48, while the bundle with the knife set and CDs sold on average for $25.44. This difference is statistically significant (independent-samples one-tail t-test = -2.31, df = 76, p = 0.02).

Next we check for superadditivity in the bundle consisting of the knife set and the DVD player. The knife set sold at an average price of $38.48, the DVD player for $50.76, while the bundle with the knife set and DVD player sold on average for $124.43. Since we used a between-subjects design, we cannot add the individual bids for the knife set and DVD player and compare this to the bundle. Therefore, we used a t-test based on the linear combinations of group means (Ramsey and Schafer 2002, p. 152-55). Results indicate significance of superadditivity, based on an independent-samples one-tail t-test = -1.69, df = 108, p < .05.

Finally, we use the same test for the bundle consisting of the DVD and CDs. For this bundle, where both items had high value certainty, there is no evidence of subadditivity, hyper-subadditivity or superadditivity. Stated more formally, we cannot reject the additivity of valuations (t-test = -0.24, df = 116, p = 0.80).

Figure 3.4 Average Bid Amounts for Experiment 2



These results provide strong support for our predictions, and are entirely consistent with our explanation of inferring item value in the formation of bundle valuation. To support this explanation more thoroughly, however, we must investigate the process measures.

Recall that our model proposes that decision makers, when determining the value of a bundle, will first estimate the value of the separate components in a bundle, usually beginning with the item for which they are most certain about value. We expected that decision makers will make inferences about the value of the uncertain item from the certain item, and therefore that decision makers will focus first on the certain item.

Participants were asked which of the items in the bundle they considered first, plus their degree of certainty concerning the value (on a 10-point scale). For the two bundles consisting of the low-certainty knife set and one of the high-certainty goods (CDs or DVD player), 82.02% of the participants (73 out of 89) considered the item they were most certain about first.[20] For the third bundle, consisting of the CDs and DVD player, with little difference in the degree of certainty in the values, most participants focused on the more expensive DVD player.

To test our hypothesis of the inference of value, we first consider the negative (positive) effect of the retail price of CDs (DVD player) on the retail price of the knife set, while controlling for the difference in the level of uncertainty in the estimates of the retail prices. For this purpose we estimate the following two regression models:

$$RP_{knife} = \alpha_i + \beta_1 RP_{cd} + \beta_2 CP_{cd\text{-}knife} + \varepsilon_i \qquad (1)$$

$$RP_{knife} = \alpha_i + \beta_3 RP_{dvd} + \beta_4 CP_{dvd\text{-}knife} + \varepsilon_i \qquad (2)$$

Where RP_{knife} = the estimated retail price for the knife set, RP_{cd} = the estimated retail price for the CDs, RP_{dvd} = the estimated retail price for the DVDs, $CP_{cd\text{-}knife}$ = difference in the level of certainty in the individuals' estimate of the retail price of CD versus the knife set, $CP_{dvd\text{-}knife}$ = difference in the level of certainty in the individuals' estimate of the retail price of DVD versus the knife set, and α_i, β_1, β_2, β_3, β_4 are the parameters to be estimated, for $i = 1, \ldots n$ bidders (α_i is an individual specific intercept, included to capture unobserved differences between individuals).

Consistent with our expectations, the estimate of the retail price for CDs has a negative effect on the retail price estimate for the knife set ($\beta_1 = -1.14$, $p < .05$), while the estimate of the retail price for the DVD player has a positive effect ($\beta_3 = .22$, $p < .01$). Furthermore, the negative effect for CDs is more substantial when the difference in the level of certainty in the individuals' estimate of the retail price of the CDs minus the retail price of the knife set is greater ($\beta_2 = -6.47$, $p < .01$). This coefficient is not significant for the difference between the DVD player and knife set ($\beta_4 = 1.91$, $p < .52$).

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[20] This is consistent with the responses to another survey question. Participants were asked their level of agreement on a 10-point scale (from strongly disagree = 0, to strongly agree = 10) to the following statements: 1. When determining the value of this bundle, I added my estimates for each of the individual items together (*M* = 6.16, *SD* = 2.86). 2. When determining the value of this bundle, I first considered the item of which I was most certain about the price (*M* = 6.81, *SD* = 2.63).

The results of these regression analyses, while consistent with our hypotheses, do not provide a definitive answer of the direction of inference. Further analyses provide more proof that the CDs retail price estimates influence those of the knives. When individuals are more certain about their estimate of the retail price of the knife set, their estimate of the retail price is \$29.95 for CDs and \$90.25 for the knife set. However, when individuals are more certain about the retail price of CDs, their estimate of the retail price is \$28.73 for CDs and \$61.80 for the knife set. While the CD estimate remains the same, the estimates for the knife set are lower ($t = 1.76$, $df = 43$, $p = 0.10$) when individuals are more certain about the retail price of CDs.

Because we concluded that retail price estimates of CDs and DVD player influenced the retail price of the knife set, we next consider the effect of the retail price on the valuation of the bundle. Since the need for the item and an individual's budget constraint have an important influence on the actual bid amount in an auction (Milgrom 2004), we will control for these effects. The item need is obtained from the survey conducted at the completion of the auction. All bidders expressed their level of need for the different items on an 11-point scale. Because we do not have a direct measure of the budget constraint, a proxy variable was created. This variable captures the differences in the bid amounts across individuals, while adjusting for their estimates of retail price and need for the item.[21]

The following two models were estimated, with the bid amount, or bidders' valuations, of the bundle as the dependent variable:

$$BV_{cd.\ knife} = \alpha + \beta_1 Budget + \beta_2 RP_{cd} + \beta_3 RP_{knife} + \beta_4 Need_{cd} + \beta_5 Need_{knife} + \beta_6 RP_{cd} \times Need_{cd} + \beta_7 RP_{knife} \times Need_{knife} + \varepsilon_i \qquad (3)$$

$$BV_{dvd.\ knife} = \alpha + \beta_1 Budget + \beta_2 RP_{dvd} + \beta_3 RP_{knife} + \beta_4 Need_{dvd} + \beta_5 Need_{knife} + \beta_6 RP_{dvd} \times Need_{dvd} + \beta_7 RP_{knife} \times Need_{knife} + \varepsilon_i \qquad (4)$$

where $BV_{cd.\ knife}$ = bid value for the bundle consisting of CDs and knife set, Budget = a proxy variable, which controls for the bidders' budget constraint, RP_{cd} = estimated retail price of the CDs, $Need_{cd}$ = participants perceived need for the CDs obtained from the survey, RP_{cd} x $Need_{cd}$ = interaction effect between retail price and need.

The results in Table 3.1 show the relationship between a participant's estimate of the retail price for each of the components of the bundle and her/his valuation of the bundle. For the bundle of CDs and knife set, only the retail value of the knife set, if the need is high ($\beta_7 RP_{knife} \times Need_{knife}$), has a significant effect on the valuation of the bundle. The valuation of the bundle consisting of the DVD and knife set is positively influenced by the retail value of the DVD player ($\beta_6 RP_{dvd} \times Need_{dvd}$) and the retail value of the knife set, if the need is high ($\beta_7 RP_{knife} \times Need_{knife}$). The budget variable is significant for the bundle consisting of the DVD player and knife set, but not for the CDs and knife set bundle. This is likely because the average bid for the DVD player and knife set bundle is more than 4 times the average bid value for the CDs and knife set bundle.[22]

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[21] For the equation consisting of the bundle with the CDs and knife set, the budget measure for bidder i = bid on DVD / (estimate of retail price of DVD x Need for DVD), while for the bundle consisting of the DVDs and knife set the measure for bidder i = Bid on CDs / (estimate of retail price of CDs x Need for CDs). This variable captures the proportion of the retail estimate bid for the other bundle. Because these two measures are highly correlated, this is a reasonable proxy variable for a bidder's budget constraint.

Table 3.1 Parameter Estimates

Bundle **CD and Knife set**	**Parameter Estimates**[a]	**Bundle** **DVD and Knife set**	**Parameter Estimates**
Intercept	13.790 (12.19)	Intercept	-0.161 (6.07)
Budget	1.066 (1.82)	Budget	50.274*** (13.41)
Retail price CDs	.084 (.33)	Retail price DVDs	-.031 (.25)
Retail price Knife set	-.149 (.10)	Retail price Knife set	-.496 (.63)
Need for CDs	2.722 (2.07)	Need for DVD	-5.64 (10.62)
Need for Knife set	-.260 (1.70)	Need for Knife set	-13.92 (10.43)
Retail price CDs x Need for CDs	-.025 (.06)	Retail price DVDs x Need for DVDs	.128** (.06)
Retail price Knife set x Need for Knife set	.042** (.02)	Retail price Knife set x Need for Knife set	.294** (.14)

[a] standard errors in parenthesis

*** Coefficients are statistically significant at the 0.01 level

** Coefficients are statistically significant at the 0.05 level

Discussion

Results of the second experiment indicate that bundling the low-value certain item (CDs) with the higher-value uncertain item (knife set) leads to hyper-subadditivity, while bundling the high-value certain item (DVD player) with the lower-value uncertain item (knife set) resulted in superadditivity. Further analyses showed significant relationships between estimates of the retail price of the high certainty item and subsequent estimates of the retail price of the uncertain item. This indicates that the low (high) value certain item negatively (positively) influenced estimates of the uncertain item. We also demonstrated that these estimates of the retail prices significantly impacted the bid for the bundle.

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[22] Results of models 3 and 4, without the budget variable, are consistent with those presented in Table 3.1.

More specifically, the results of Experiment 2 provide evidence of both superadditivity and hyper-subadditivity. On average, participants bid 51% more for the knife set alone than for the bundle consisting of the knife set and the CDs. Superadditivity was also observed for the bundle of the knife set and the DVD player: average bids for the bundle were 39% higher than the sum of the average bids of the separate auctions for the knife set and the DVD player. These results are in sharp contrast to those of previous researches, which demonstrated superadditivity only in bundling for items that are highly functionally complementary (e.g., Harlam et al. 1995).

In this paper we argue that subjects draw inferences concerning the value of an uncertain item based on the value of a certain item. However, it may be possible that subjects draw inferences about the quality of the knife set from their value estimates of the other products in the bundle. To consider this option we look at the quality ratings of the knife set for different groups of subjects (see Table 3.2). Quality rating for the products are based on the question "How do you rate the quality of this product?", where 0 = very low quality and 10 = Very high quality. On average the quality ratings are the lowest for the CD's and the highest for the DVD. Recall that all subjects participated in two auctions for the three products, one for a bundle consisting of two of the products and a single auction for the third product. In Table 3.2, group 1 consists of subjects who bid on the bundle consisting of the CDs and the knife set and a single auction for the

DVD, group 2 subjects bid on the bundle consisting of the DVD and the knife set and a single auction for the CDs, and group 3 subjects bid on the bundle consisting of the CDs and DVD and a single auction for the knife set.

Table 3.2 Survey Results for Experiment 2

Ratings for Knives					
Group	**Quality**	**Certainty Quality**	**Retail Value**	**Certainty Retail Value**	**Need**
1. CDs & Knife set	6.60	4.20	67.44	3.51	3.51
2. DVD & Knife set	6.11	4.24	78.56	3.96	4.51
3. CDs and DVD	6.48	4.26	75.07	4.12	3.31
Ratings for DVD					
Group	**Quality**	**Certainty Quality**	**Retail Value**	**Certainty Retail Value**	**Need**
1. CDs & Knife set	7.64	6.89	151.77	5.73	3.91
2. DVD & Knife set	7.80	7.00	152.14	5.80	4.09
3. CDs and DVD	7.07	6.25	142.97	5.77	4.07
Ratings for CD's					
Group	**Quality**	**Certainty Quality**	**Retail Value**	**Certainty Retail Value**	**Need**
1. CDs & Knife set	5.31	5.04	28.80	5.22	3.56
2. DVD & Knife set	5.70	5.54	27.34	5.02	4.93
3. CDs and DVD	5.72	5.37	30.83	6.00	4.42

If subjects draw quality inferences from the value of the other good in the bundle we would expect the quality ratings for the knife set to be highest when bundled with the DVD (group 2) and to be lowest when bundled with the CDs (group 1). This is clearly not the case, since the quality rating for the knife set was 6.11 when bundled with the

DVD and 6.60 when bundled with the CDs. The quality ratings from the different groups for the knife set are not statistically significantly different (the same it true for the quality ratings for the CDs and the DVD). Similarly, we can not reject equality of subject's certainty about the quality for different groups of subjects for the different products. This is based on responses to the following question: "How **certain** are you about the **quality** of this product?", where 0 = very uncertain and 10 = very certain. Finally, subject's estimates of the local retail value, and their certainty concerning this estimate were not statistically significant for the different groups, for each of the products. These estimates were based on the corresponding questions from the survey in Appendix 3.1.

We also explored the mechanisms underlying the inference value observed. Specifically, we found that decision makers do in fact focus on the more certain item in the bundle, and they do so regardless of the importance or price. In total, 82% of the participants first considered the item they were most certain about. Together, these results provide strong support for our explanation of value inference under conditions of value uncertainty. A final test of our conceptual model would be to eliminate both hyper-subadditivity and superadditivity by reducing the uncertainty about the knife set. This issue is addressed in Experiment 3.

3.4.3 Experiment 3

The objective of the third experiment is to test our hypothesis that decision makers use the value of high-certainty goods to infer the value of uncertain goods. In Experiment 2 we bundled an item of uncertain value with a certain item of either a lower value or a higher value. As hypothesized, we found that the high (low) value item with high certainty provides a positive (negative) indicator about the value increasing (decreasing) the valuation of the low certain item. If these results are due to value inferences when bundling high-value certainty goods with low-value certainty goods, then we should be able to reduce or eliminate them by increasing the certainty of the value associated with the uncertain item. Therefore, in Experiment 3 we reduce the uncertainty of the value of the uncertain item used in Experiment 2, by providing the average retail value of this item.

As in the first two experiments, we used a Vickrey auction to determine the participants' valuations. The same three bundles were used as in Experiment 2, with the exception that we provided the local average retail price for the uncertain good (the knife set). Therefore, we directly manipulate the degree of uncertainty within item (the knife set), between experiments 2 and 3. For the local retail price, we used the average estimated retail price from Experiments 1 and 2.

Note that providing an estimate of the average local retail price will reduce the degree of uncertainty, while not completely eliminating uncertainty, as retail prices still vary across outlets. Again we used a mixed design, where each participant bid on all three items, but in one of three different bundle groupings. They participated in two auctions: one for a single item, and another for a bundle consisting of the two other items. After finishing the auctions, participants completed a survey about all items in the auctions.

Results

One hundred and fifty-six students participated in this study, of which 130 placed meaningful bids in the auctions. A summary of the results of Experiment 3 is provided in Figure 3.5.[23] In contrast to the first two experiments, these results are not consistent with hyper-subadditivity when the low value (high certainty) CDs are bundled with the medium value (high certainty) knife set. Actually, the bundle of the knife set and the CDs sold at an average price of $24.48, which is slightly higher than the sum of the separate auctions ($22.48). When the high value (high certainty) DVDs are bundled with the medium value (high certainty) knife set, the bundle sells at a higher price than the sum of the separate components. For the CDs and the DVD players, the bundle price is slightly lower than the sum of the separate components. However, none of these differences are statistically significant (see Table 3.3). Therefore, we cannot reject additivity of valuations of the bundle components for any of the three bundles.

Table 3.3 Tests for Additivity of Components

Item	Sum Separate Components	Bundle	T-values	Probability
CDs & Knife set	22.49	24.48	0.46	0.65
DVD & Knife set	55.48	69.95	1.20	0.23
CDs and DVD	52.73	49.23	-0.38	0.71

[a]Results are based on the tests described in Experiment 2

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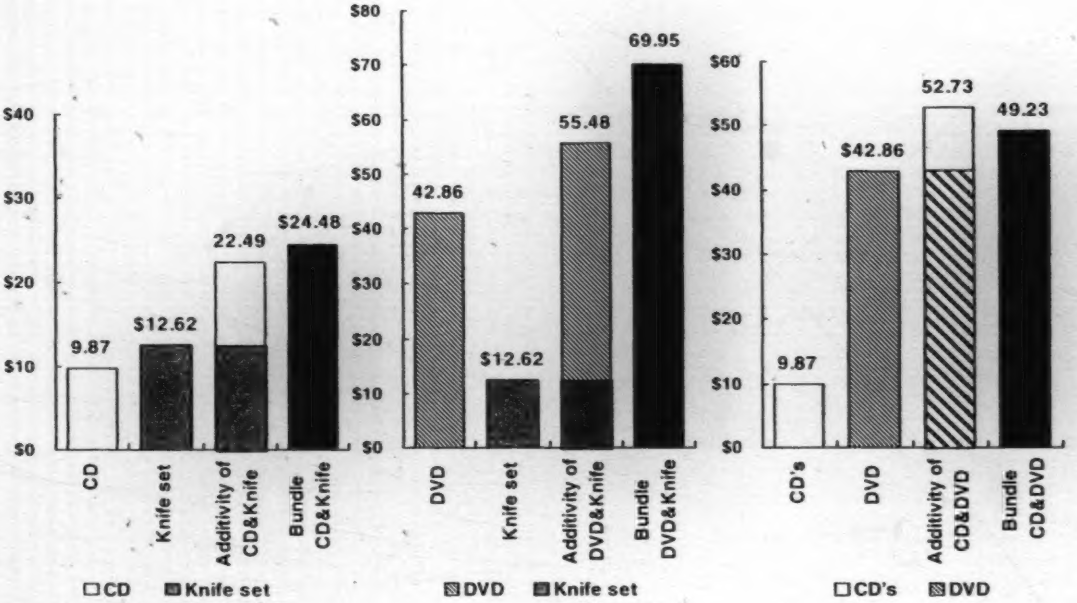
[23] The average bid amount for the knife set in Experiment 3 is considerably lower from the average values in Experiments 1 and 2. Because we specified an average retail price of $75, the value distribution is truncated at this level in Experiment 3 (different from Experiments 1 and 2). All bids were well below $75.

Figure 3.5 Average Bid Amounts for Experiment 3



Discussion

In the third experiment we wished to determine whether the effects of hyper-subadditivity persist when the uncertainty about the value of goods is reduced. The results show that the hyper-subadditivity effect is no longer present when decision makers are informed of the value of the uncertain item (the knife set). In all instances, we could not reject additivity of the bundle valuation, as the valuations for all three bundles are equal to the sum of the values of the two component goods. Hence, we have shown further strong support for the critical role of value uncertainty in our previous findings of both hyper-subadditivity and superadditivity. That is, when bundle components are neither complements nor substitutes, and when the values of all components are relatively certain, bundle valuations are additive. When value uncertainty is reduced, bidders behave as expected by basic economic theory.

3.5 General Discussion

The results of the three experiments provide strong support for our framework. We observe hyper-subadditivity in both study one and study two. We also observe the opposite, superadditivity, in study two. Self reports of bidders are consistent with our framework, as are the results of our regression analyses. Specifically, decision makers tend to use the more certain item in a bundle to make inferences about the value of other goods in a bundle. Finally, and also consistent with our framework, departures from additivity are eliminated when value uncertainty is reduced.

We now turn to a discussion of potential shortcomings of our method and conceptualization. First we address the appropriateness of the uncertain value item to our bidders. We then address the difference between our conceptualization and extant explanations for departures from additivity.

Many readers may question the relevance/importance/appropriateness of a set of kitchen knives to university undergraduates. It should be pointed out that this research was carried out at a publicly funded university, with mostly third and fourth year students. The majority of these students live in shared accommodations (i.e. multiple students sharing an apartment or house) where they prepare their own food. In fact, this item was initially selected for pretesting because it had been selling well on a local, student oriented Internet auction site. In addition, the data themselves speak to the fact that this was an item of use or interest to our bidders.

Across the three studies, 91.9% of participants bid on this item in the knives only condition. This is similar to the overall level of bidding on CD's (90.8%), and DVD players (92.8%). Furthermore, the willingness-to-pay measures in all three experiments indicate that subjects, on average, are willing to pay a significant amount for knife sets in isolation (Mean = $23.82). In addition, measures of need for the knife set were only slightly lower than for the other two items (in experiments 2 and 3). Over 50% of subjects in experiments 2 and 3 did bid on the knife for their own usage, while the others bid on the knife set for a gift.

Of course, future research should attempt to replicate these results with other item classes, and under other circumstances. We chose to focus on bundling items that were neither complements nor substitutes, specifically because we wanted to make sure that our results were free from the impact of complementarity and substitutability. Since we have now demonstrated that it is possible to obtain superadditivity without complementarity, and (hyper-) subadditivity without substitutability, future research may now wish to explore how complementarity and substitutability interact with inferences about value. We now turn to concerns about alternative explanations for our results.

The astute reader will recognize that our inference based conceptualization shares some key attributes with extant explanations for similar phenomena, most notably anchoring and adjustment and the evaluability hypothesis. A close consideration of our results, however, should make it clear that neither of these explanations adequately explain our data.

Anchoring and adjustment has been used in the bundling literature to explain subadditivity and superadditivity. Yadav (1994), for example, proposed that decision makers anchor their valuations on one of the components of the bundle and either over adjust or under adjust from this value, leading to superadditivity and subadditivity respectively. This explanation is inconsistent with several aspects of our results. Yadav (1994) proposed and found that decision makers anchor (focus) on the most important/expensive item in the bundle. Our bidders focused on the more certain item in the bundle, regardless of price[24]. This difference alone makes it unlikely that the specific anchoring and adjustment mechanism proposed by Yadav would explain our results.

More importantly, however, our bidders were also asked to report the process they used to arrive at their bundle valuation. The vast majority indicated that they assessed the value of each individual item, and then added them together. Hence, the process does not appear to involve anchoring and adjustment in any traditional sense (i.e., anchors on one item and directly adjusts from that anchor to arrive at the bundle valuation). In addition, we demonstrate through our regression analysis that the value of the certain item has a direct effect on the perceived retail price of the uncertain item. This is supportive not only of our framework; it is also supportive of the veracity of the self reports about bundle valuation process. Taken together, it seems much more likely that our bidders were engaged in a process of "inferring then adding" rather than anchoring and adjustment.

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[24] Because Yadav's important/expensive bundle components were also likely to be the most certain, one could, in fact, argue that his findings are a special case of ours.

The evaluability hypothesis (Hsee 1996; González-Vallejo and Moran 2001) also appears, at first glance, to be a reasonable alternative explanation for our findings. While the evaluability hypothesis can explain preference reversals in joint versus separate evaluations, this only occurs when certain important attributes are difficult to evaluate in isolation, but easy to evaluate in side-by-side comparisons. In other words, the evaluability hypothesis relies not upon ambiguity about a specific attribute level, but rather on the ambiguity about the relative merit of a clearly stated attribute level. None of the items we used exhibited attribute levels that were clearly stated, but non-evaluable in isolation.

To reiterate, clearly there was ambiguity about the value of the knives in our study. If one cares to view overall value as an attribute, then the knives can be said to have possessed ambiguity with regard to the level of this attribute. But this ambiguity was about attribute level, and not the merit of a particular stated level with respect to some benchmark. Because of this, the evaluability hypothesis is also not an appropriate explanation for our findings.

Future Research

In order to test our theory, we selected two items that were neither substitutes nor complements. While such bundles are frequently observed in the real world (e.g., a free television comes with the purchase of a dining room set), future research should also consider the bundling of complementary or substitute items. Non-complementary of items make it more difficult to get superadditivity and non-substitutes make it harder to get subadditivity. This makes the hyper-subadditivity and superadditivity reported in this paper more robust. Finally, research results should be generalized to different types of items and categories.

Although the anchor and adjustment hypothesis proposed by Yadav (1994) can not explain the observations reported in this study, one may argue that people may anchor on the value of more certain product (CD or DVD) and make adjustment to less certain product (the set of knives) when there is uneven certainty in the values of products in a bundle, and people may anchor on the value of more expensive product in a bundle when they are equally certain about the value of individual products, as Yadav (1994) proposed. While this hypothesis does not sound very likely, admittedly it is also consistent with our observation. In further study, it will be of interest to further investigate the exact psychological mechanism underlying values of bundles.

Further studies should also look at the boundary conditions of hypersubadditivity or superadditivity of value of product bundle. Examples of these conditions could be as follows: 1) products are sold by different sellers at same store or website vs. in different store or website; 2) products are sold in separate auctions by the same seller at same store.

3.6 Conclusion

One of the fundamental tenets of economic theory is that people have positive marginal value for goods. Hyper-subadditivity is, in principle, inconsistent with this fundamental precept. If the values about which a decision maker is certain are not simply anchors (or non-informative heuristic values) then these values are, in fact, usable information that rational actors can and should employ in forming bundle valuations.

Other studies have reported hyper-subadditivity, but the mechanisms offered therein were quite different from our value inference explanation. We view the value inference explanation to be highly parsimonious, as it can account for many previous findings as well as those reported here. We believe this shift in conceptualization is an important one, as it provides not only a better understanding of underlying processes, but also results in the development of simpler models that make specific predictions about decision maker's bundle valuations.

People should not prefer less to more. They certainly should not be willing to pay more for less. By introducing a value inferencing process to the less-is-more phenomenon, it has become possible envision how decision makers, behaving in a mindful and rational way, can prefer less to more, and even pay more for less.

Appendix 3.1: Instruction and Quiz

Important Instructions for Sealed-Bid auctions

We would like to present you with an unique opportunity to potentially get an extremely good deal on several product. You will be able to participate in **two** second-price sealed-bid auctions for several different products. In a sealed-bid auction you can only place one bid per auction (so TWO in total). The winner will be the bidder with the **highest bid** and the price paid is the price of the **second highest bid**! For example, consider the following five top bids: Michael will be the winner, but he will only pay $90.

Bidder	**Michael**	Donna	Peter	Marry	Paul
Bid amount	$100	**$90**	$84	$80	$78

All auctioned items will be sold to the highest bidder regardless of the final price. There will be several different auctions and you will be randomly assigned to one of the auctions for the product on which you bid. There will be approximately 10 participants in each auction.

Informed consent: By continuing beyond this point, you agree to participate in a study being conducted by researchers at the University of Alberta.

Your responses will be treated as strictly confidential, analyzed in anonymous form only, and used for the sole purpose of academic research. You will be asked to provide your name and e-mail address. This information is collected for the sole purpose of notifying the winners. Once this has been done, all personal information will be destroyed. Please note that you may participate in this study only once.

Once a bid has been submitted, it is **binding**. Do **not** bid any more than what you are willing to pay for the product. The auction winners will be expected to **pay** the amount of their successful bid upon delivery of the product, which will be in about one week. Acceptable forms of payment are personal checks, or cash. Winners will be contacted by e-mail.

Please note that these auctions are not designed to entice you to buy a product that you do not want. You are, of course, free to not submit any bid at all in your auction. However, because of the small number of bidders, it is likely that the winner of each auction will end up purchasing the product at a fraction of its market value. Therefore, please view this is an opportunity for you to potentially get an extremely good deal on a product.

Toshiba DVD Player

TOSHIBA
SD-430V

BRAND-NEW, FACTORY-SEALED, Toshiba SD-430V single disk DVD Player (Silver colour)

- Progressive Scan Dolby Digital and DTS Digital outputs for Superior Color Detail and Resolution
- Natural Digital Audio Quality - 24 bit / 192 kHz Audio DAC for Superior Digital Clarity and Warmth
- Multi-Format Playback - DVD / DVD-R / CD / CD-R / CD-RW / Video CD / MP3 / WMA/ Digital Picture CD
- Digital Photo Viewer, 4x Picture Zoom
- Included In Box: cables, Battery, Remote, Manual.
- Renowned Toshiba quality, with 1-Year warranty

Bid Amount $ ___________

CD-R Media 50 CD Spindle 80 min & New 16 piece Cutlery set

50 CD-R

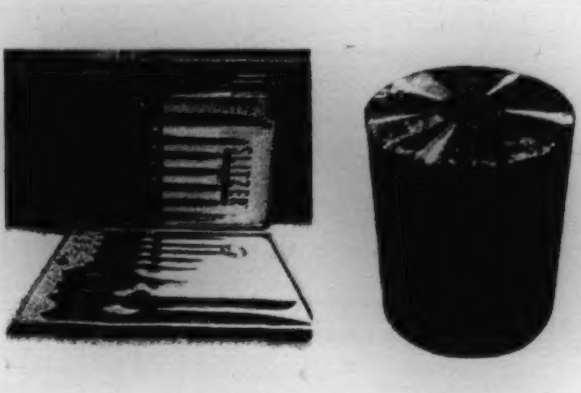
Spindle

700 MB
24 x certified
Made in India

Average Retail price in Edmonton $70.00
Professional Style Cutlery
Dishwasher Safe
Durable Leymar Handle Surgical Stainless Steel Blade
Includes: Ham/Bread Knife, 8 Steak Knife, Carving Knife, Chef Knife, Boning Knife, Scissors, and Sharpening steel, etc.
Limited Lifetime Warranty

Bid Amount $ ________________

Please answer the following questions related to the auctions you just completed.



(a). In the auction with the two products, which of the two items did you think about first (please check only one)?

_____ The CD-R Media 50 CD Spindle 80 _____ The 16 piece Cutlery set

Please put an X on the lines below, indicating the level of agreement with the following statements.

(b). When determining the value of this bundle, I added my estimates for each of the individual products together.

0______1______2______3______4______5______6______7______8______9______10
Strongly disagree agree nor disagree Strongly agree

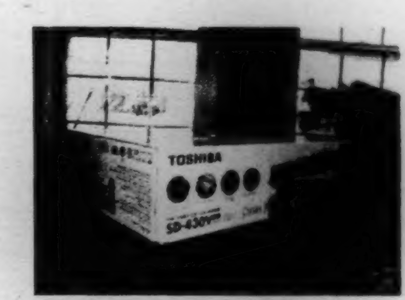
(c). When determining the value of this bundle, I first considered the product of which I was most certain about the price

0______1______2______3______4______5______6______7______8______9______10
Strongly disagree agree nor disagree Strongly agree

(d). When determining the value of this bundle, I first considered the most expensive product.

0______1______2______3______4______5______6______7______8______9______10
Strongly disagree agree nor disagree Strongly agree

For DVD Player



Please answer the following questions

(a) How do you rate the **quality** of this product?

0______1______2______3______4______5______6______7______8______9______10
Very low quality Neutral Very high quality

(b) How **certain** are you about the **quality** of this product?

0______1______2______3______4______5______6______7______8______9______10
Very Uncertain Neutral Very certain

(c) How much do you expect this product will cost at a store in Edmonton?

$____________________ Local retail price

(d) How **certain** are you about your estimate of the **local retail price** of this product?

0______1______2______3______4______5______6______7______8______9______10
Very Uncertain Neutral Very certain

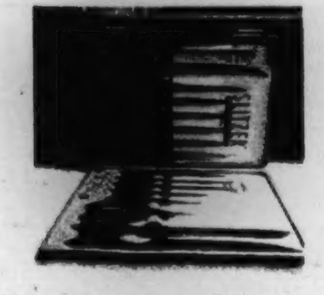
(e) How do you rate your **need** for this product?

0______1______2______3______4______5______6______7______8______9______10
Don't need it at all Neutral Need it very much

(f) How do you rate your degree of expertise of this product?
0______1______2______3______4______5______6______7______8______9______10
I know nothing Neutral I am an expert

(g) Do you bid on this product for own usage or for gift?
________own usage ________gift

For New 16 piece Cutlery set



Please answer the following questions

(a) How do you rate the **quality** of this product?

0_____1_____2_____3_____4_____5_____6_____7_____8_____9_____10
Very low quality Neutral Very high quality

(b) How **certain** are you about the **quality** of this product?

0_____1_____2_____3_____4_____5_____6_____7_____8_____9_____10
Very Uncertain Neutral Very certain

(c) How do you rate your **need** for this product?

0_____1_____2_____3_____4_____5_____6_____7_____8_____9_____10
Don't need it at all Neutral Need it very much

(d) How do you rate your degree of expertise of this product?

0_____1_____2_____3_____4_____5_____6_____7_____8_____9_____10
I know nothing Neutral I am an expert

(e) Do you bid on this product for own usage or for gift?

________own usage ________gift

For The CD-R Media 50 CD Spindle 80

Please answer the following questions

(a) How do you rate the **quality** of this product?

0______1______2______3______4______5______6______7______8______9______10
Very low quality Neutral Very high quality

(b) How **certain** are you about the **quality** of this product?

0______1______2______3______4______5______6______7______8______9______10
Very Uncertain Neutral Very certain

(c) How much do you expect this product will cost at a store in Edmonton?

$____________________ Local retail price

(d) How **certain** are you about your estimate of the **local retail price** of this product?

0______1______2______3______4______5______6______7______8______9______10
Very Uncertain Neutral Very certain

(e) How do you rate your **need** for this product?

0______1______2______3______4______5______6______7______8______9______10
Don't need it at all Neutral Need it very much

(f) How do you rate your degree of expertise of this product?

0______1______2______3______4______5______6______7______8______9______10
I know nothing Neutral I am an expert

(g) Do you bid on this product for own usage or for gift?

________own usage ________gift

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Chapter 4

Essay 3: An Empirical Comparison of Three Auction Strategies for Multiple Products

4.1 Introduction

Auctioneers often have multiple complementary products to auction off. Complementarity is present when bidders' value of the bundle is higher than the sum of individual values of these products. Complementarity may be due to savings in transaction costs when one wins multiple products. For example, a winner of two eBay auctions run by the same auctioneer may save on shipping costs if the shipment can be combined. Complementarity may also be a consequence of extra utility from consuming the products together. A typical example is a set of antique furniture for which people usually are willing to pay more to have the complete set. An additional reason for people to pay more than the sum of the individual values is that it will take tremendous effort (if possible at all) to find all the individual pieces from different sellers.

We will consider an auctioneer selling two different products using second price sealed-bid auctions (also called Vickery auctions). In a Vickery auction it is optimum for bidders to bid their value.[25] An auctioneer, selling one unit each of two products, A and B, typically has three alternative selling strategies:

(1) One auction for the bundle consisting of A and B

(2) Two simultaneous separate auctions for A and B

(3) Two sequential separate auctions for A and B

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[25] In the remainder of the paper we assume that sellers use a Vickery auction and that bidders will bid their value.

For the seller, each of the three strategies has its advantages and disadvantages. When complementary is not present, separate auctions are efficient because winners are always the bidders with the highest values for the individual products. This efficiency minimizes the consumer surplus the winners have and therefore increases the revenues of separate auctions. However, when complementarity is present, bidders in separate auctions find themselves facing a so called exposure risk (Bykowsky et al. 1995; Ausubel et al. 1997; Rothkopf et al. 1998; Chakraborty 2004). This is the risk of winning only one of the products at a price higher than the individual values, or winning both products at a price higher than the value of the bundle, as bidders may bid above their value(s) in an attempt to win both auctions and receive the extra complementarity.[26] Therefore, strategic bidders, in separate auctions, may bid less aggressively due to the exposure risk, reducing the profitability of separate auctions.

The exposure risk affects bidders differently in simultaneous and sequential auctions for two complements, say product A and product B. In two sequential Vickery auctions, bidders only face exposure risk in the first auction. In the second auction losing bidders' weakly dominant strategy is to bid the value of B, while the winner of A will bid an amount equal to the sum of the value of B and the complementarity. In the first auction, bidders may overbid to try to win A to increase their chance to win both auctions and receive the complementarity.[27] Therefore a bidder may win the first product by overpaying and lose the second product. In simultaneous auctions, bidders run a potential exposure risk in both auctions, as they may overbid in both auctions to increase their chance of winning the complementarity.

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[26] In certain instances, it may even occur when winning both products by paying a price higher than the value of the bundle plus the complementarity.

[27] Subramanian and Venkatesh (2004) showed that it is an optimum strategy for bidders to bid above their value for the first product in a sequential auction to increase the chance of winning both products.

In a bundled auction, bidders do not face this exposure risk and can bid more aggressively, adding the entire complementarity to their bid. This lack of exposure risk may translate to higher revenue. However, an auction for the bundle is generally inefficient because the winner is the bidder with the highest value for the bundle, and not necessarily the bidder with the highest values for the individual products. This inefficiency decreases the revenue of a bundled auction.

Simply put, when complementarity is present, in two separate auctions the efficiency increases the revenue while the exposure risk decreases the revenue. In an auction of the bundle, the inefficiency decreases the revenue but the lack of exposure risk increases the revenue. Therefore, when choosing a selling mechanism, an auctioneer of the two complements faces a tradeoff between the inefficiency of a bundled auction and the exposure risk problem in two separate auctions, and the optimality of each selling strategy depends on the net effect of these two mechanisms.

The primary objective of this empirical study is to compare the profitability of these three selling mechanisms under different conditions. The environment is characterized by (1) the number of bidders (N), (2) heterogeneity of bidders' individual values of the two products, and (3) complementarity of the two products (C). Another objective is to find out how the environment affects bidders' strategies for these three selling mechanisms. Specifically, we are interested in looking at how the environment affects bidders' perceived exposure risk and how the exposure risk affects bidders' overbidding in separate auctions. These issues are examined through both analytical models and laboratory experiments. We first derive the Bayesian Nash equilibrium strategy and next compare the theoretical predictions with actual decisions by subjects, to determine whether they behave as theory predicted.

To the best of our knowledge, this study is the first in the auction literature to

- Use an analytical model to investigate how strategic bidders bid under different conditions defined by the number of bidders, complementarity and heterogeneity of bidders' values for each of the three auction mechanisms,
- Compare the revenues of all the three typical selling mechanisms under different conditions,
- Empirically test the result of the model in a controlled laboratory experiment.

The remainder of this paper is as follows. We start with a review of the literature. In section 3 the model and several propositions are discussed. The details and results of the experiment are reported in section 4. We conclude the paper with a discussion of the key findings.

4.2 Literature Review

4.2.1 Bundling Literature in a Posted Price Context

Bundling, as a pervasive selling mechanism, is defined as "the sale of two or more separate products in one package" (Stremersch and Tellis 2002), where "separate products" means products for which separate markets exist. It is a widely used marketing practice, to sell a wide variety of products, including seasonal tickets for sports events, high speed Internet and cable TV, air tickets, hotel and car rentals.

Research on bundling as a pricing mechanism was initiated by Stigler (1968). Since then bundling has received considerable attention by academics in the field of economics (Adams and Yellen 1976; Schmalensee 1984; McAfee, McMillan, and Whinston 1989; Salinger 1995) and marketing (Guiltinan 1987; Gaeth etc. 1990; Yadav 1994, 1995; Yadav and Monroe 1993; Bakos and Brynjolfsson 1999, 2000; Soman and Gourville 2001; Stremersch and Tellis 2002; Jedidi et al. 2003).

Bundling has been shown to increase sellers' profits by permitting more complete extraction of buyers' residual consumer surplus. This is because bundling can reduce the heterogeneity of buyers' reserve prices, by serving as a second-degree price discrimination mechanism (Ex. Adams and Yellen 1976; Schmalensee 1984). While most previous research has only considered asymmetry, Stremersch and Tellis (2002), propose that there are two dimensions in the distribution of reservation prices: asymmetry and variation. In their survey of the economics and marketing literatures on bundling, they find that ambiguity exists concerning the concept of heterogeneity of reservation prices. They argue that the distribution of reservation prices consists of asymmetry and variation, and correlation alone is not sufficient to represent heterogeneity. Asymmetry refers to the difference among consumers' reservation prices for the separate products. For two separate products A and B, asymmetry occurs when one segment of buyers has a relatively higher reservation price for A, while the other segment has a higher reservation price for B. Variation means the difference among consumers' reservation prices for the bundle of products. Asymmetry leads to negative correlation while variation leads to positive correlation. Stremersch and Tellis (2002) show that these two dimensions affect the optimality of bundling in different ways, and, hence, it is important to incorporate both aspects of heterogeneity. The result of the first essay also finds that both asymmetry and variation must be considered when we compare the revenues of a bundled auction vs. separate auctions -since heterogeneity of values among bidders can not be represented by the correlation of individual values when complementarity is present.

Besides heterogeneity of values, complementarity[28] of multiple products has been shown to affect the profitability of bundling (Lewbel 1985; Matutes and Regibeau 1988, 1992; Telser 1979, Guiltinan 1997, Venkatesh and Kamakura 2003). Venkatesh and Kamakura (2003) find that the optimality of different selling mechanisms (unbundled sales, pure bundling, and mixed bundling) is determined by the degree of complementarity. For example, when marginal cost is low, pure bundling is optimal for moderate-to-strong complements and mixed bundling is optimal for independently valued products and weak complements.

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[28] Oxenfeldt (1966) identified eight important sources of complementary of demand: One-Stop Shopping; Impulse buying; Broader Assortment; Related Use; Enhanced Value; Prestige Builder; Image Effects; Quality Supplements relationships. Guiltinan (1997) categorizes complementary into three types: saving purchasing time and effort; enhancing satisfaction with other products; enhancing image of the seller so all products are valued more highly.

Examining the existing bundling literature, we identify heterogeneity of consumer's reservation prices (values) and the degree of complementarity as two key factors deciding the profitability of bundling. Therefore, we will incorporate both heterogeneity of values and complementarity in this study.

4.2.2 Auction Literature for Multiple Objects Auctions

Although most auction studies have focused on individual product auctions, auction of multiple products is a very active area of research (see Klemperer 2004 for a review). Prior economics studies have examined optimal auction design for multiple products (e.g. Maskin and Reiley 1984, Armstrong 2000, Levin 1997, Avery and Hendershott 2000), simultaneous auctions (e.g., Wilson 1979, Anton and Yao 1992, Krishna and Rosenthal 1996), sequential auctions (e.g. Bernhardt and Scoones 1994, McAfee and Vincent 1997, Jeitschko 1999) and combinatorial auctions (see Milgrom 2004 for a review). The multiple products can be either homogeneous (Wilson 1979, Krishna and Rosenthal 1996) or heterogonous (Palfrey 1983, Chakraborty 1999, Levin 1997). The topic has also begun to receive attention from marketing researchers (for example, Zeithammer 2005; Cheema et al. 2005; Subramanian and Venkatesh 2004).

One track within the multiple product auction literature has looked at comparing three typical selling mechanisms for multiple products in term of profitability, as follows:

1) Bundled auction vs. Simultaneous auctions. Palfrey (1983) compares the profitability of one bundled Vickrey auction versus two simultaneous separate Vickrey auctions and shows that when there are only two bidders, the bundled auction is more

profitable than separate auctions. Based on Palfrey (1983)'s framework, Chakraborty (1999) finds that for two products whose values are independently distributed, there is a threshold for the number of bidders above which separate auctions will always be more profitable. So in general these two papers have concluded that without complementarity simultaneous separate auctions are more profitable than bundled auctions for more than two bidders. Both studies assume that bidders' values of the component products are independently distributed and there is no complementarity.

2) Bundled auction vs. Sequential auctions. Subramanian and Venkatesh (2004) examined the profitability of one auction for the bundle versus two sequential auctions for two complementary products. They conclude that when complementarity is small and there are more than four bidders, separate auctions are more profitable. However, when complementarity is moderate or large, a bundled auction is always more profitable. Although their conclusions are in part based on the assumption that the individual values of the two products are independently distributed.

3) Simultaneous auctions vs. Separate auctions. Krishna & Rosenthal (1996) argue that for two complements, simultaneous and separate auctions are approximately equally profitable. However, their results are limited due to the very strict assumptions made.[29] Hausch (1986) compared simultaneous and sequential auctions for two affiliated value identical products. He identified two opposing effects in sequential auctions: (i) when bids are announced between auctions, they may convey information about the

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[29] They assume that there are only two kinds of bidders; local and global bidders. A global bidder has equal values for these multiple products and for her the value of the bundle exceed the sum of the individual values, while a local bidder wants only one of the products and received no complementarity for winning both products. It is not clear if their conclusions apply when global bidders have unequal values for the individual products.

values of products to be sold later on. which increase the revenues (an information effect); (ii) bidders who are aware of the information effect tend to bid lower in the first auctions and therefore reduce revenue (a deception effect). The optimality of sequential auctions depends on the net effect of these two effects. Feng and Chatterjee (2002) look at a seller who has multiple identical products to sell to N bidders who arrive sequentially and only want one unit of the product. They indicate that the ratio of the number of items to the number of bidders decides whether sequential auctions are more profitable or not. When the ratio of the number of bidders to the number of items for sale is below a threshold value, sequential auctions have higher expected revenue than simultaneous auctions.

While auctioning off multiple products with complementarity is of significant managerial importance, a close examination of the literature reveals several significant gaps. First of all, there is no study comparing all three mechanisms showing under what conditions sellers should sell products in a bundle or sell them in separate auctions (either simultaneously or sequentially). Second of all, surprisingly given the importance of heterogeneity in buyers' values in bundling literature, heterogeneity has been largely overlooked and it is not clear how the two elements --- variation and asymmetry--- affect the revenue of these selling mechanisms. Last but not least, no empirical studies have tested the theoretical predictions of these models. The current study is aiming to bridge these gaps in the auction literature. Table 4.1 summarizes the differences between the current study and previous studies.

Based on the literature review above, our study is innovative for studying the following in multiple product auctions:

- Revenue comparison of three auction mechanisms: Bundled auction, simultaneous separate auctions, and sequential separate auctions.
- Empirical investigation, using a controlled laboratory experiment
- Number of bidders
- Complementarity
- Heterogeneity of individual values

Table 4.1 Summary of Studies Comparing Auction Mechanisms for Multiple Products

Study	Palfrey (1983)	Hausch (1986)	Krishna & Rosenthal (1996)	Chakraborty (1999)	Feng & Chatterjee (2002)
Auctions	Vickrey, Bundled vs. Separate[30]	Vickrey, Sequential vs. Simultaneous	Vickrey, Combinatorial, Sequential, Simultaneous	Vickrey, Bundled vs. Separate	Vickrey, Sequential vs. Simultaneous
Products	Private value Different products	Affiliated value Identical products	Private value Identical products	Private value Different products	Private value Identical products
Complementarity	No	No	Yes	No	No
Number of bidders	≥2	2	≥2	≥2	≥2
Heterogeneity of Bidders Values	Independently distributed	Positively correlated	Global bidders and Local bidders	Independently distributed	Positively correlated
Bidder's Strategies	No	Yes	Limited	No	Yes
# of Products	≥2	2	2	2	2
Theoretical / empirical	Theoretical	Theoretical	Theoretical	Theoretical	Theoretical
Conclusions	Only when the number of bidders is small, bundling is more profitable than separate auctions.	Optimality of sequential auctions depends on winner's curse	Simultaneous auctions are slightly more profitable than sequential auctions, but much more profitable than combinatorial auctions	When the number of bidders is greater than a unique critical number, unbundling is preferred	When the ratio of the number of bidders to the number of items for sale, is below a threshold value, sequential auctions have higher expected revenues

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[30] In Palfrey (1983) and Chakraborty (1999) there is no complementarity so simultaneous and sequential auctions are essentially the same.

Table 4.1 Continued

Study	Subramanian & Venkatesh (2004)	Current Study
Auctions	Vickrey, Bundled vs. Sequential	Vickrey, Bundled, Simultaneous, Sequential
Products	Private value Different products	Private value Identical/Different products
Complementarity	Yes	Yes
Number of bidders	≥2	≥2
Heterogeneity of Bidders Values	Independently distributed	Both Variation and Symmetry
Bidder's Strategies	Yes	Yes
# of Products	2	2
Theoretical / empirical	Theoretical	Theoretical / empirical
Conclusions	Sequential auctions are optimal even for moderate complements as long as there are at least a few bidders	When there is only variation among bidders' values, the three mechanisms are equally profitable. When there is only asymmetry, separate auction are more profitable when the number of bidder is large.

4.3 Theory

4.3.1 The Auctions

A revenue maximizing auctioneer has one unit of two products A and B, which can be either identical or different, to sell to N bidders ($N \geq 2$). These two products are to be auctioned by one of the following three auction mechanisms,

1) One Vickrey auction[31] for the bundle consisting of products A and B. Each bidder submits just one bid (b_{bu}) for the bundle. The bidder with the highest b_u wins, and the price the winner pays equals the second highest bid[32].

2) Two simultaneous separate Vickrey auctions. Each bidder submits two bids (b_A, b_B) respectively for products A and B. In each of the two auctions, the bidder with the highest bid wins, and the price the winner pays equals the second highest bid. The winners are announced simultaneously; hence, when placing a bid on one product, they don't know the outcome of the other auction.

3) Two sequential separate Vickrey auctions, with the first auction for product A followed by a second auction for product B[33]. Each bidder first places a bid (b_A) for product A, followed by a bid for product B, which is conditional on the outcome of

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[31] A major reason for using a Vickrey auction is that without complementarity bidders' weakly dominant strategy is to bid value for the products, regardless of their risk attitude or the number of competing bidders. Most previous research has also used Vickrey auctions (e.g. Palfrey 1983; Krishna and Rosenthal 1996; Chakraborty 1999; and Subramanian and Venkatesh 2004).

[32] The following rules are employed to handle ties. In the bundled auction, when there is a tie, the high bidders (say *m* bidders) have an equal chance (i.e. chance of $1/m$) to win the bundle. In the two separate auctions, suppose there are *m* highest bidders on A, *n* highest bidders on B, and *j* highest bidders on both A and B. Please note that the *j* bidders are the overlap of the *m* and *n* bidders. When $j=0$, each of *m* bidders has a chance of $1/m$ to win A and each of *n* bidders has a chance of $1/n$ to win B; when $j>0$, each of *j* bidders has a chance of $1/j$ to win both A and B.

[33] As we will show later, A and B have same individual distribution, so the order of selling does not matter.

the auction for product A ($b_{B|\text{outcome } A}$). In both auctions, the bidder with the highest bid in each auction wins and pays a price equal to the second highest bid in that auction. [34]

The following assumptions are made in this paper:

- Both seller and bidders are risk-neutral,

- The number of bidders (N) is the same in the two separate auctions and in the bundled auction. The number of bidders is common knowledge to all bidders and to the seller,

- Complementarity (C) for products A and B is the same for all bidders, regardless of their individual values of A and B (V_A and V_B). C is common knowledge to all bidders and the seller[35]. C is standardized in the same units as V_A and V_B, where $0 < C < \infty$,

- A bidder's value of the bundle of A and B (V_{bu}) equals the sum of her individual values of A and B (V_A and V_B) and the complementarity,

- Each bidder's V_A and V_B are privately known and are realizations of the same distribution that is common knowledge to all bidders and the seller.

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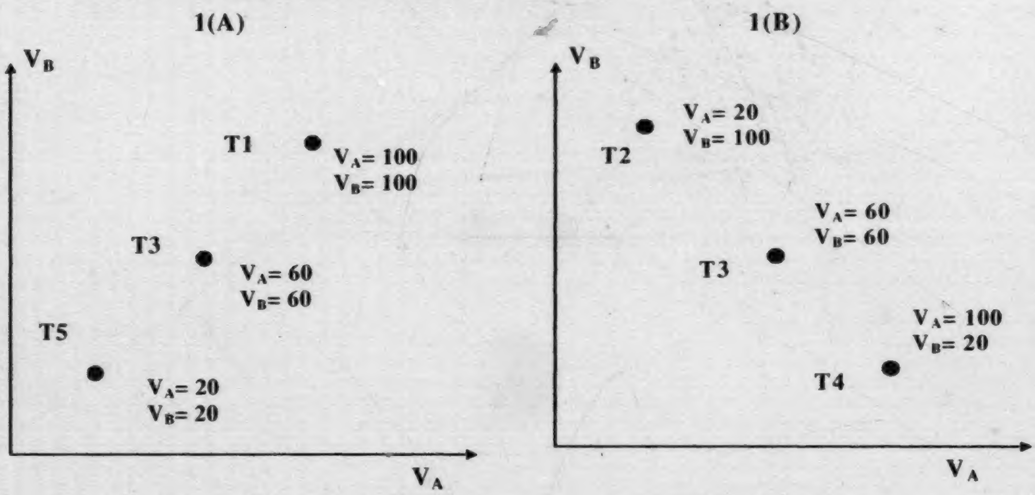
[34] This paper does consider combinatorial auction as an alternative selling mechanism. Porter et al. (2003) argue that there are three major reasons for rarity of combinatorial auctions in practice. (1) Computational uncertainty: The selection of the winning bids and what it would cost for competition to displace them typically requires the solution of integer-programming problems. And there is no guarantee that the solution for such a problem can be found in a ``reasonable`` amount of time when the number of bidders and items becomes larger. (2) Bidding complexity: Combinatorial auctions would be burdensome and difficult for participants and the auctioneer. This is because that least there are inconceivably many packages on which a bidder might want to place bids, and selecting any subset may be strategically awkward and provide the auctioneer with incomplete information. Also there is a computational problem for the bidder to determine how much to bid to be successful. (3)Threshold problem: Krishna and Rosenthal (1996) show that this threshold problem makes combinatorial auctions less profitable than sequential and simultaneous auctions. Suppose each of two small bidders is bidding on a separate item, but a third bidder is bidding on a package that contains both items. Then the two small bidders must implicitly coordinate through their bidding to ascertain what price each will pay in order for the sum of both bids to exceed the package bid.

[35] Here we examine the heterogeneity of individual values V_A and V_B, not complementarity. In many cases, while people may have different individual values of two products, they have similar value of the complementarity (see the FCC auctions and eBay auctions examples mentioned before).This assumption is also made by Krishna and Rosenthal (1996).

In this study there are two distributions of bidders' valuations (shown in Figure 4.1), in each there are three bidder segments (types) with three different combinations of V_A and V_B. Each bidder has one third chance of being chosen by nature to be of one of the three potential types. In the first distribution (see Figure 4.1(A)), V_A and V_B of the three types T1, T3, and T5 are respectively ($100, $100), ($60, $60), and ($20, $20). In the second (see Figure 4.1(B)) V_A and V_B of the three types T2, T3, and T4 are ($20, $100), ($60, $60), and ($100, $20) respectively. In distribution 1, there is only variation and no asymmetry in bidders' individual values, and, hence, the two values are perfectly positively correlated. There is only asymmetry and no variation in distribution 2, implying that the two values are perfectly negatively correlated. Therefore a main advantage of adapting these two distributions is that we can look at the effect of each of the two dimensions of heterogeneity in bidders' individual values while controlling the other.

Figure 4.1 Joint Distributions of Individual Values of Two Products



1(A)

1(B)

We set the number of bidders at either 2 or 10[36]. Given that Palfrey (1983), Chakraborty (1999), and Subramanian & Venkatesh (2004) identify two, three and four as the threshold number of bidders to decide the relative profitability of bundled auction, we believe 2 bidders is low and 10 is high. We choose C=20 and C=50 as low and high levels of complementarity.

Therefore we obtain eight (2×2×2) different combinations (scenarios) of the heterogeneity of bidder's two values (distribution 1 and 2 in Figure 4.1), the number of bidders N (2, 10) and the level of complementarity C (20, 50), as shown in Table 4.2.

Table 4.2 Eight Combinations (Scenarios) of Individual values, N and C

Scenario No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Distribution of Values	1st	1st	1st	1st	2nd	2nd	2nd	2nd
T1 (V_A=100, V_B=100)	yes	yes	yes	yes				
T2 (V_A=20, V_B=100)					yes	yes	yes	yes
T3 (V_A=60, V_B=60)	yes	yes	yes	yes	yes	yes	yes	yes
T4 (V_A=100, V_B=20)					yes	yes	yes	yes
T5 (V_A=20, V_B=20)	yes	yes	yes	yes				
Number of Bidders	2	2	10	10	2	2	10	10
Complementarity	20	50	20	50	20	50	20	50

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[36] Previews studies show monotonicity in the effect of number of bidders on the relative profitability of bundled vs. separate auctions. For example, Palfrey (1983), Chakraborty (1999) and Subramanian & Venkatesh (2004) all show there exists a threshold for number of bidders. When the number of bidders is greater than this threshold, separate auctions are more profitable. The first essay confirms this finding. Therefore we consider two levels of number of bidders in this study.

We will start by deriving the results of the analytical model, as follows. For each combination for the number of bidders N, complementarity C and distribution of V_A and V_B, bidders come up with their Bayesian Nash Equilibrium bids in each of the three auction mechanisms. The seller, with the knowledge of the number of bidders, the distribution of bidders' types and all the type contingent bids, calculates and compares the expected revenues of the three selling mechanisms. The objective of this study is to find out under what conditions which of the three selling mechanisms is most profitable.

4.3.2 The Equilibrium Strategies

4.3.2.1 One Auction for the Bundle

For an auction of the bundle consisting of A and B, the following result follows from our assumptions.

Proposition 1*: In a Vickrey auction for the bundle, a bidder's weakly dominant bid equals her value for the bundle* V_{bu}, *i.e.,* (V_A+V_B+C), *where* V_A *and* V_B *are the bidder's individual values of A and B, and C is the value of complementarity.*

Proof: The bundled auction is actually an auction for just one product, i.e., the bundle. Therefore, it is optimum for a bidder to bid her value of the bundle (see Krishna 2002).

The type contingent bids of all types in bundled auction are shown in Table 4.3(A). In all auctions of the bundle, bidders add the entire C to their bids, regardless of N and the distribution of V_A and V_B.

4.3.2.2 Two Simultaneous Separate Auctions

In two simultaneous auctions, each bidder submits two bids b_A and b_B and her expected surplus is given by:

$$\pi = P_{AB} \cdot (V_A + V_B + C - b_{Ah} - b_{Bh}) + P_A \cdot (V_A - b_{Ah}) + P_B \cdot (V_B - b_{Bh})$$

where P_{AB} is the probability of winning both A and B

P_A is the probability of winning A and losing B

P_B is the probability of winning B and losing A

b_{Ah} is the 2nd highest b_A (submitted by other bidders)

b_{Bh} is the 2nd highest b_B (submitted by other bidders)

In each of the two auctions, bidders may overbid to increase their chance of winning both products and the complementarity. However, there is a chance of winning only one product and paying a price higher than the value of this product, or winning both products and paying a price higher than the value of the bundle. This exposure risk may prevent people from bidding aggressively and reduces the revenue.

For two simultaneous separate auctions, the following results can be drawn from the assumptions.

Proposition 2*: In two simultaneous auctions, bidders' type contingent Bayesian Nash Equilibrium bids for the eight possible scenarios are as specified in Table 4.3(B).*

Proof: See the Appendix 4.4.

Table 4.3(B) shows that in two simultaneous auctions, the effect of competition (through N) on bidders' type contingent strategies (specifically how much of C is added to b_A and b_B) depends on the heterogeneity of bidders' values V_A and V_B.

In the first four scenarios where only variation is present in bidders' V_A and V_B, which are perfectly positively correlated, bidder's type contingent bids are not affected by the number of bidders. Each type of bidder adds 0.5C to their b_A and b_B. For each bidder the sum of two bids equals her value of bundle (V_A+V_B+C). The highest bids from her opponents b_{Ah} and b_{Ah} are either both higher than (with the exception of T1 bidders), or both equal to, or both lower than (with the exception of T5 bidders) her two bids. So there is no chance of losing one product while winning the other, or winning both and paying a price higher than her value for the bundle. Only when both b_{Ah} and b_{Ah} are lower than her two bids, her expected profit is not zero.

When only asymmetry is present, and bidders' V_A and V_B are perfectly negatively correlated, the number of bidders has a significant affect on bidders' strategies.

When there are two bidders and C is 20 (scenario 5), for each bidder, the sum of two bids equals the value of the bundle [V_A+V_B+C]. T3 bidders add half of the complementarity to their two bids, while T2 and T4 bidders put all C on the bids for the low value products. When C increases to 50 (scenario 6), T2 and T4 bidders again put all C on the bids for the low value products; however, T3 bidders have a different asymmetric type contingent strategy. One T3 bidder adds more than C to her two bids (40 and 40) to outbid T2 and T4 bidders, while the other T3 bidder only adds 10 dollars to each bid. Therefore when C increases from 20 to 50, the sum of T3 bidders' two (average) bids increases by 30.

When there are ten bidders, T3 bidders do not take C into full account due to the exposure risk. When C is 20 (scenario 7), T3 bidders have a different asymmetric type contingent strategy. One T3 bidder adds half C to each of her two bids, while the other T3 bidders bid their values. When C increases to 50 (scenario 8), all T3 bidders add only 10 dollars to each of the two bids. As a result, when C increases from 20 to 50, the sum of T3 bidders' two (average) bids increases by 18, indicating higher exposure risk when N=10.

Table 4.3(A) BNE Bids in One Auction for the Bundle

Scenario	1: (N=2, C=20)	2: (N=2, C=50)	3: (N=10, C=20)	4: (N=10, C=50)	5: (N=2, C=20)	6: (N=2, C=50)	7: (N=10, C=20)	8: (N=10, C=50)
T1 (V_A=100, V_B=100)	b_{bu}=220	b_{bu}=250	b_{bu}=220	b_{bu}=250				
T2 (V_A=20, V_B=100)					b_{bu}=140	b_{bu}=170	b_{bu}=140	b_{bu}=170
T3 (V_A=60, V_B=60)	b_{bu}=140	b_{bu}=170	b_{bu}=140	b_{bu}=170	b_{bu}=140	b_{bu}=170	b_{bu}=140	b_{bu}=170
T4 (V_A=100, V_B=20)					b_{bu}=140	b_{bu}=170	b_{bu}=140	b_{bu}=170
T5 (V_A=20, V_B=20)	b_{bu}=60	b_{bu}=90	b_{bu}=60	b_{bu}=90				

Table 4.3(B) BNE Bids in Two Simultaneous Auctions

Scenario	1: (N=2, C=20)	2: (N=2, C=50)	3: (N=10, C=20)	4: (N=10, C=50)	5: (N=2, C=20)	6: (N=2, C=50)	7: (N=10, C=20)	8: (N=10, C=50)
T1 (V_A=100, V_B=100)	b_A=110 b_B=110	b_A=125 b_B=125	b_A=110 b_B=110	b_A=125 b_B=125				
T2 (V_A=20, V_B=100)					b_A=40 b_B=100	b_A=70 b_B=100	b_A=40 b_B=100	b_A=70 b_B=100
T3 (V_A=60, V_B=60)	b_A=70 b_B=70	b_A=85 b_B=85	b_A=70 b_B=70	b_A=85 b_B=85	b_A=70 b_B=70	b_{A1}=100, b_{B1}=100 b_{A2}=70, b_{B2}=70*	b_{A1}=70, b_{B1}=70 b_{Ai}=60, b_{Bi}=60 where i=2…10	b_A=70 b_B=70
T4 (V_A=100, V_B=20)					b_A=100 b_B=40	b_A=100 b_B=70	b_A=100 b_B=40	b_A=100 b_B=70
T5 (V_A=20, V_B=20)	b_A=30 b_B=30	b_A=45 b_B=45	b_A=30 b_B=30	b_A=45 b_B=45				

*b_{Ai} and b_{Bi} mean bidder *i*'s b_A and b_B (i=1… N).

4.3.2.3 Two Sequential Separate Auctions

The BNE bids in two sequential auctions can be solved by backward induction. In the second auction of the two sequential auctions. for the winner of A the marginal surplus of winning B is equal to the difference between (V_B+C) and the second highest bid on B. Clearly, her optimal bid on B ($b_{B|winA}$) is equal to (V_B+C). For the loser of A, the optimal bid on B ($b_{B|loseA}$) is equal to V_B since there is no chance of winning the complementarity.

For the first auction for A, each bidder's expected surplus by bidding (b_A, $b_{B|winA}$ or $b_{B|loseA}$) is given by

$$\pi = P_{12} \cdot (V_A + V_B + C - b_{Ah} - b_{Bh}) + P_1 \cdot (V_A - b_{Ah}) + P_2 \cdot (V_B - b_{Bh})$$

Where P_{12} is the probability of winning A with b_A and winning B with $b_{B|winA}$

P_1 is the probability of winning A with b_A and losing B with $b_{B|winA}$

P_2 is the probability of losing A with b_A and winning B with $b_{B|loseA}$

b_{Ah} is the 2nd highest b_A (submitted by other bidders)

b_{Bh} is the 2nd highest b_B (submitted by other bidders)

The knowledge of winning A in the first auction will increase the marginal surplus of winning B. Therefore, strategic bidders may overbid in the first auction for A. However, when a bidder overbids in the first auction, there is a chance that she wins A but does not win B, and overpays for A. Knowing this risk, in the first auction a bidder faces a tradeoff between bidding high to win both products or bidding lower to avoid the exposure risk.

For two sequential auctions for A and B, the following result follows from our assumptions.

***Proposition 3**: In two sequential auctions, bidders' type contingent Bayesian Nash Equilibrium bids for the eight possible scenarios are as specified in Table 4.3(C).*

Proof: See the Appendix 4.4.

In two sequential auctions, the effect of competition on bidders' type contingent strategies (specifically how much of C is added to b_A) also depends on the heterogeneity of bidders' V_A and V_B.

In scenarios 1 to 4, where only variation is present in bidders' V_A and V_B, bidder's type contingent bids are not affected by the number of bidders. In the first auction for A, all bidders add the entire C to their b_A. This will be the case, since there is no chance of losing one product while winning the other because the winner of A will also win B. Hence, the lack of exposure risk induces bidders to bid more aggressively in the first auction.

When only asymmetry between V_A and V_B is present and there are two bidders (scenario 5 and 6), each bidder type adds the entire C to their b_A. However, when there are ten bidders (scenario 7 and 8), T4 and T3 bidders, even though they have a good chance of winning A, add very little C to their b_A, since they have very little chance of winning B (for which T2 bidders have higher values and bids). Hence increased competition increases the exposure risk, and bidders will bid less aggressively.

The difference between two (in scenario 5 and 6) versus ten bidders (in scenario 7 and 8) is that T4 or T3 bidders have no exposure risk when there are only two bidders (there is no chance that they will win only A, at a price above V_A). For example, in scenario 6, for a T4 bidder, if her only opponent is also T4, she has 50% chance of winning A by paying (V_A+C) and winning B with a margin of profit of C, and 50% chance of losing both auctions. Therefore, in either case her profit is zero. If her opponent is T3, she wins A and loses $10 in the first auction and wins B and wins $10 in the second auction, and again her profit is zero. If her opponent is T2, she wins only A and has a profit of $30. However when N=10, bidding above their value on A is risky for T4 and T3 bidders. For example, suppose all T4 bidder add θ (θ>0) to their b_A. For a T4 bidder, there is a chance that some of her opponents are T4 and some T2; therefore she has a positive chance of winning A (and paying θ higher than her V_A) and losing B. Knowing this exposure risk, T4 and T3 bidders lower their b_A when N=10.

Table 4.3(C) BNE Bids in Two Sequential Auctions

Scenario	1: (N=2, C=20)	2: (N=2, C=50)	3: (N=10, C=20)	4: (N=10, C=50)	5: (N=2, C=20)	6: (N=2, C=50)	7: (N=10, C=20)	8: (N=10, C=50)
T1 (V_A=100, V_B=100)	$b_A=120$ $b_{B\|WinA}=120$ $b_{B\|LoseA}=100$	$b_A=150$ $b_{B\|WinA}=150$ $b_{B\|LoseA}=100$	$b_A=120$ $b_{B\|WinA}=120$ $b_{B\|LoseA}=100$	$b_A=150$ $b_{B\|WinA}=150$ $b_{B\|LoseA}=100$				
T2 (V_A=20, V_B=100)					$b_A=40$ $b_{B\|WinA}=120$ $b_{B\|LoseA}=100$	$b_A=70$ $b_{B\|WinA}=150$ $b_{B\|LoseA}=100$	$b_A=40$ $b_{B\|WinA}=120$ $b_{B\|LoseA}=100$	$b_A=70$ $b_{B\|WinA}=150$ $b_{B\|LoseA}=100$
T3 (V_A=60, V_B=60)	$b_A=80$ $b_{B\|WinA}=80$ $b_{B\|LoseA}=60$	$b_A=110$ $b_{B\|WinA}=110$ $b_{B\|LoseA}=60$	$b_A=80$ $b_{B\|WinA}=80$ $b_{B\|LoseA}=60$	$b_A=110$ $b_{B\|WinA}=110$ $b_{B\|LoseA}=60$	$b_A=80$ $b_{B\|WinA}=80$ $b_{B\|LoseA}=60$	$b_A=110$ $b_{B\|WinA}=110$ $b_{B\|LoseA}=60$	$b_A=60+x3^{*}$ $b_{B\|WinA}=80$ $b_{B\|LoseA}=60$	$b_A=60+x3'^{*}$ $b_{B\|WinA}=110$ $b_{B\|LoseA}=60$
T4 (V_A=100, V_B=20)					$b_A=120$ $b_{B\|WinA}=40$ $b_{B\|LoseA}=20$	$b_A=150$ $b_{B\|WinA}=70$ $b_{B\|LoseA}=20$	$b_A=100+x4^{*}$ $b_{B\|WinA}=40$ $b_{B\|LoseA}=20$	$b_A=100+x4'^{*}$ $b_{B\|WinA}=70$ $b_{B\|LoseA}=20$
T5 (V_A=20, V_B=20)	$b_A=40$ $b_{B\|WinA}=40$ $b_{B\|WinA}=20$	$b_A=70$ $b_{B\|WinA}=70$ $b_{B\|WinA}=20$	$b_A=40$ $b_{B\|WinA}=40$ $b_{B\|WinA}=20$	$b_A=70$ $b_{B\|WinA}=70$ $b_{B\|WinA}=20$				

*b_A is the expected value of a mixed strategy. x3, x4, x3' and x4' are random variables satisfying continuous distributions whose cumulative distribution functions are defined in the proof in Appendix 4.2, where $0 \le x3 \le 20$, $0 \le x4 \le 20$, and $0 \le x3' \le 50$, $0 \le x4' \le 50$. The expected values of x3, x4, x3' and x4' are respectively 0.007, 0.00015, 10.003 and 0.13.

4.3.3 Comparison of Revenues

For each auction mechanism in each of the eight scenarios, the expected revenue equals the weighted sum of the revenues for all potential outcomes. See Appendix 4.3, 4.4 and 4.5 for the calculations of the expected revenues of the three selling mechanisms. Table 4.4 summarizes the expected revenues of each selling mechanism in the eight scenarios as predicted by theory.

In scenarios 1, 2, 3 and 4 where there is only variation among V_A and V_B, which are perfectly positively correlated, the three selling mechanisms are equally profitable. The reason is twofold. First, there is no inefficiency in bundled auctions since the winner is the bidder with the highest values of the bundle (V_A+V_B+C), who also has the highest V_A and V_B. Second, there is no exposure risk in the Nash equilibria for the two separate auctions since a bidder either wins both products or loses both. Therefore, N and C affect the revenues of the three mechanisms to the same extent.

When there is only asymmetry among bidders' V_A and V_B, hence they are negatively correlated, none of the three mechanisms strictly dominates the others based on profitability. It is more profitable to sell two complements in a bundle when N=2, especially when C=20 (scenario 5). However, when N=10 (scenario 7 and 8) separate auctions are more profitable. The two separate auction mechanisms generate approximately the same revenues, while sequential auctions have slightly higher revenues when N=2 and simultaneous have higher revenues when N=10. When N=10, exposure risk has a significant effect on the revenue of separate auctions, which can be demonstrated by comparing scenarios 7 and 8. When C increases from 20 to 50, the revenue of bundled auctions increases by 30, while the revenues of both simultaneous and sequential auctions remain virtually unchanged!

Table 4.4 Theoretical Expected Revenues in Eight Scenarios

Scenario		1	2	3	4	5	6	7	8
Number of Bidders		2	2	10	10	2	2	10	10
Complementarity		20	50	20	50	20	50	20	50
Heterogeneity	Variation	Yes	Yes	Yes	Yes	No	No	No	No
	Asymmetry	No	No	No	No	Yes	Yes	Yes	Yes
Bundled Auction		104.47	134.47	211.66	241.66	140	170	140	170
Simultaneous Auctions		104.47	134.47	211.66	241.66	113.31	153.3	192.45	194.93
Sequential Auctions		104.47	134.47	211.66	241.66	117.75	163.30	191.67	193.59

4.4 Experiment

4.4.1 Experiment Design

The bidders were 68 undergraduate business students at a major North American university. Participants were provided with detailed instruction and were shown an example of a second-price, sealed-bid auction.

We explained to them that

(1) They would attend a series of auctions and bid on two hypothetical products, products A and B. Each participant would be provided with a value for each of the two products.

(2) The values of these products are drawn from one of the two distributions demonstrated in Figure 4.1. In each auction, each participant was told the specific distribution from which her and her rivals' values for products A and B were drawn.

(3) In each auction, the winner of an auction will obtain an amount equal to the difference between her value for the product and the amount of the second-highest bid. Each bidder has 100 "e-dollars" in her account (Each e-dollar equals one cent). All gains (losses) from auctions in this study will be added to (subtracted from) subjects' accounts.

(4) Whenever a bidder wins both A and B, she gets an extra bonus, which represents the complementarity between two products.[37]

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[37] In the auctions of the bundle of A and B, the bonus was given to the winner of the bundle. In two separate auctions, only winners of both A and B obtain the bonus.

(5) They were told the number of opponents they will compete against in each auction.

To help bidders understand the concept of a Vickrey auction, we conducted one practice run of a Vickrey auction for a hypothetical product. The outcome of the auction was revealed. Next all participants completed a short quiz about a Vickrey auctions and the correct answers were announced. Finally, bidders entered the real experimental auctions.

Each bidder is required to bid in all eight scenarios for each of the three auction mechanisms. Thus, the experiment employed a four-factor (auction mechanism, N, C and distribution of values), twenty four-level (scenario) within-subject design.

Auctions using the same mechanisms were always put in the same block. So there are three blocks (mechanisms) with eight scenarios in each, and there are six possible orders for the three blocks. The order of blocks (mechanisms) was randomized, as well as the order of the eight scenarios within each block.

In each of the eight scenarios, subjects were told the number of opponents they competed against, the distribution from which their opponents' values were drawn, and the amount of complementarity for the two products. In each scenario there were three (pairs of) auctions, in each (pair of) auction a subject was given a pair of values, V_A and V_B (one out of the three in the given distribution) and was required to bid on each auction. Thus in a scenario defined by N, C and a distribution of V_A and V_B, we have each bidder's bids for each of the three pairs of values. Participants were told that only one of the three auctions would actually be conducted. For example, in a scenario where N=2, C=20 and V_A and V_B are drawn from distribution 2, a bidder participated in three auctions in which her values are respectively (\$20, \$100), (\$60, \$60) and (\$100, \$20). The bidder was told to place bid in each of the three auction based on these values, while her opponent's values could be any of the three pairs with equal chance. Only one auction was executed to determine the bidder's profit (for this scenario).

In an auction for the bundle, each subject was asked to place one bid. In two simultaneous separate auctions, each subject placed one bid on A and one on B. In two sequential separate auctions, a bidder was required to submit one bid for the first product A, and submit two bids for B; one if she were to win A ($b_{B|WinA}$) and one if she were to lose A ($b_{B|LoseA}$). Bidders did not know the outcome of the first auction when they bid in the second auction.

To make sure that bidders understood the rules of each selling mechanism before the real auctions in each block (selling mechanism), two practice rounds were run and outcomes were shown for demonstration purposes. This was followed by a short quiz with several questions about the selling mechanism. These quizzes served as filters for each of the selling mechanism (experimental blocks). In the following data analysis for each mechanism, we only include the bids from the subjects who correctly answered all questions on the quiz about this mechanism.[38] One typical session lasted about 75 minutes.

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[38] Out of 68 subjects, respectively 59. 55 and 55 subjects correctly answered all questions about the auctions for the bundle, the simultaneous auctions and the sequential auctions. Four subjects failed all three quizzes and four subjects failed two quizzes.

4.4.2 Result: Bids

The average bids are summarized in Table 4.5(A)-(C) for the three selling mechanisms.

4.4.2.1 One Bundled Auctions

In all auctions of the bundle, bidders' bids are approximately equal to their corresponding values for the bundle (V_A+V_B+C), regardless of N and heterogeneity of values. As theory predicts, the bids on the bundle increase as C increases, and an increase in N has little impact on bids[39].

4.4.2.2 Two Simultaneous Auctions

When only variation is present and V_A and V_B are perfectly positively correlated (scenarios 1 to 4), comparison of all types of bidders' bids in scenarios 1 and 2 and comparison of bids in scenario 3 and 4 reveal that people increase their two bids to the same extend when C increases from 20 to 50, regardless of N, as theory predicts.

In scenarios 5 to 8, where only asymmetry is present, T2 and T4 bidders do not change their bids according to N, while T3 bidders bid less aggressively due to the exposure risk. A comparison of scenarios 5 and 6 shows that, when N=2, T3 bidders increase the sum of their two bids by $21.63 (paired t=5.12, df=54, p=.00) when C increases from 20 to 50. However, when N=10, T3 bidders actually decrease the sum of their two bids by $2.39 (paired t=-0.283, df=53, p=.778) when C increases from 20 to 50.

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[39] The only exception is the T5 bidder's bid in scenario 3 (t=2.877, df =58, p= .005).

Table 4.5 (A) Average Bids in One Auction for the Bundle

Scenario	1: N=2, C=20	2: N=2, C=50	3: N=10, C=20	4: N=10, C=50	5: N=2, C=20	6: N=2, C=50	7: N=10, C=20	8: N=10, C=50
T1 (V_A=100, V_B=100)	b_{bu}=212.76	b_{bu}=240.95	b_{bu}=219.56	b_{bu}=243.36				
T2 (V_A=20, V_B=100)					b_{bu}=139.00	b_{bu}=168.64	b_{bu}=140.95	b_{bu}=170.22
T3 (V_A=60, V_B=60)	b_{bu}=138.22	b_{bu}=171.46	b_{bu}=139.85	b_{bu}=168.59	b_{bu}=139.80	b_{bu}=169.29	b_{bu}=140.58	b_{bu}=169.17
T4 (V_A=100, V_B=20)					b_{bu}=139.54	b_{bu}=168.80	b_{bu}=141.19	b_{bu}=170.46
T5 (V_A=20, V_B=20)	b_{bu}=62.71	b_{bu}=95.07	b_{bu}=72.61	b_{bu}=96.68				

Table 4.5 (B) Average Bids in Two Simultaneous Separate Auctions

Scenario	1: N=2, C=20	2: N=2, C=50	3: N=10, C=20	4: N=10, C=50	5: N=2, C=20	6: N=2, C=50	7: N=10, C=20	8: N=10, C=50
T1 (V_A=100, V_B=100)	b_A=115.52 b_B=114.75	b_A=131.29 b_B=129.34	b_A=115.49 b_B=116.09	b_A=130.34 b_B=126.98				
T2 (V_A=20, V_B=100)					b_A=40.19 b_B=103.90	b_A=62.96 b_B=116.06	b_A=37.73 b_B=106.94	b_A=62.49 b_B=109.32
T3 (V_A=60, V_B=60)	b_A=73.33 b_B=71.38	b_A=86.49 b_B=84.42	b_A=73.60 b_B=72.96	b_A=84.26 b_B=82.34	b_A=71.75 b_B=70.88	b_A=85.28 b_B=78.98	b_A=69.67 b_B=69.96	b_A=68.79 b_B=68.45
T4 (V_A=100, V_B=20)					b_A=105.62 b_B=39.73	b_A=116.74 b_B=64.34	b_A=108.21 b_B=37.83	b_A=109.57 b_B=61.19
T5 (V_A=20, V_B=20)	b_A=34.77 b_B=33.21	b_A=46.60 b_B=44.90	b_A=33.09 b_B=31.57	b_A=44.29 b_B=42.21				

Table 4.5 (C) Average Bids in Two Sequential Separate Auctions

Scenario	1: N=2, C=20	2: N=2, C=50	3: N=10, C=20	4: N=10, C=50	5: N=2, C=20	6: N=2, C=50	7: N=10, C=20	8: N=10, C=50
T1 ($V_A=100$, $V_B=100$)	$b_A=122.13$ $b_{B\|WinA}=120.49$ $b_{B\|LoseA}=102.27$	$b_A=142.89$ $b_{B\|WinA}=139.25$ $b_{B\|LoseA}=103.73$	$b_A=120.65$ $b_{B\|WinA}=123.58$ $b_{B\|LoseA}=101.55$	$b_A=142.18$ $b_{B\|WinA}=140.69$ $b_{B\|LoseA}=101.02$				
T2 ($V_A=20$, $V_B=100$)					$b_A=39.24$ $b_{B\|WinA}=113.81$ $b_{B\|LoseA}=97.64$	$b_A=58.56$ $b_{B\|WinA}=135.20$ $b_{B\|LoseA}=100.00$	$b_A=37.13$ $b_{B\|WinA}=116.87$ $b_{B\|LoseA}=95.85$	$b_A=56.35$ $b_{B\|WinA}=135.42$ $b_{B\|LoseA}=100.00$
T3 ($V_A=60$, $V_B=60$)	$b_A=75.51$ $b_{B\|WinA}=79.69$ $b_{B\|LoseA}=62.44$	$b_A=98.69$ $b_{B\|WinA}=98.56$ $b_{B\|LoseA}=63.53$	$b_A=74.91$ $b_{B\|WinA}=76.65$ $b_{B\|LoseA}=60.76$	$b_A=97.55$ $b_{B\|WinA}=98.49$ $b_{B\|LoseA}=63.00$	$b_A=77.11$ $b_{B\|WinA}=78.31$ $b_{B\|LoseA}=61.24$	$b_A=104.04$ $b_{B\|WinA}=99.98$ $b_{B\|LoseA}=63.64$	$b_A=70.05$ $b_{B\|WinA}=79.05$ $b_{B\|LoseA}=59.73$	$b_A=77.80$ $b_{B\|WinA}=101.40$ $b_{B\|LoseA}=63.36$
T4 ($V_A=100$, $V_B=20$)					$b_A=119.15$ $b_{B\|WinA}=51.87$ $b_{B\|LoseA}=23.73$	$b_A=144.02$ $b_{B\|WinA}=69.42$ $b_{B\|LoseA}=24.18$	$b_A=110.51$ $b_{B\|WinA}=50.25$ $b_{B\|LoseA}=23.73$	$b_A=111.09$ $b_{B\|WinA}=66.05$ $b_{B\|LoseA}=24.67$
T5 ($V_A=20$, $V_B=20$)	$b_A=35.58$ $b_{B\|WinA}=38.78$ $b_{B\|WinA}=22.82$	$b_A=59.00$ $b_{B\|WinA}=60.74$ $b_{B\|WinA}=23.98$	$b_A=32.98$ $b_{B\|WinA}=40.98$ $b_{B\|WinA}=23.49$	$b_A=58.24$ $b_{B\|WinA}=60.17$ $b_{B\|WinA}=23.50$				

These results indicate that T3 bidders' bids are more sensitive to exposure risk than T2 and T4 bidders'. This is because T2 and T4 bidders have a high value for one product and therefore have a good chance to win this product without adding any complementarity to the bid, while T3 bidders have median values for both products so they have to add complementarity to both products to win them both.

4.4.2.3 Two Sequential Auctions

In the second auction of the two sequential auctions, all types of bidders' $b_{B|lose\ A}$ are very close to their corresponding V_B and $b_{B|win\ A}$ to (V_B+C), regardless of N and the distribution of their opponents' values.

When only variation exists in bidders' V_A and V_B (scenarios 1 to 4), in the first auctions, all types of bidders' b_A are affected by C but not by N. As the theory predicts, when only asymmetry exists in bidders' V_A and V_B, N has little impact on T2 bidders' b_A. When C increases from 20 to 50, T2 bidders increase their b_A as much when N=2 (scenarios 5 and 6) as when N=10 (scenarios 7 and 8)[40]. N has a significant impact on T3 and T4 bidders' b_A. A comparison of scenarios 5 and 6 shows that when N=2, T3 bidders increase their b_A by $26.93 when C increases from 20 to 50. However, when N=10, T3 bidders only increase their b_A by $7.75 when C increases from 20 to 50[41]. T4 bidders increase their b_A by $24.87 from scenario 5 to 6 (when N=2) and only $0.58 from scenario 7 to 8[42] (when N=10).

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[40] 19.32 vs. 19.22, paired t=.034, df =54, p=.937.

[41] 26.93 vs. 7.75, paired t=6.008, df =54, p=.000.

[42] 24.87 vs. 0.58, paired t=6.808, df =54, p=.000.

4.4.3 Comparison of Revenues

Table 4.6 summarizes the mean of the revenue for each selling mechanisms in each of the eight scenarios. Mean revenue in each scenario was calculated by bootstrapping. In each iteration, we randomly choose N (N=2 or 10) bidders from all the bidders who attended the auctions in this scenario. For each bidder chosen, we randomly choose one of the three types and the type contingent bids. Based on these N (pairs of) bids, the revenue is decided according to the rule of each selling mechanism.

Based on the results provided in Table 4.6, we summarize how N, C and heterogeneity affect revenues of the three selling mechanisms.

1) Number of bidders. For all of the three mechanisms for given C and heterogeneity, larger N leads to higher revenues. In separate auctions, on the one hand, a larger N leads to less aggressive bidding (due to increased exposure risk), but, on the other hand, it results in a higher likelihood of having bidders with higher product values. The net effect of N on revenues is positive in separate auctions.

2) Complementarity. Generally, a higher C leads to higher revenues. There are two exceptions. A comparison of scenario 7 and 8 (N=10) shows that when C increases from 20 to 50, the revenue of two sequential auctions increases only by \$5.8 dollars (z=.27, p=.3936, one tailed), while the revenue of two simultaneous auctions actually decreases by \$2.66 dollars (z = -.12, p = .452, one tailed). In these two cases, bidders added little C to their bids due to the increased exposure risk.

3) Heterogeneity of values. For all three mechanisms, the effect of heterogeneity on revenues depends on N. When there are two bidders, asymmetry of values generates higher revenues. When there are ten bidders, variation of values leads to higher revenues.

The optimality of the three mechanisms depends on the combination of the three factors discussed above. When V_A and V_B are perfectly positively correlated and only variation is present (scenario 1, 2, 3 and 4), the three selling mechanisms are approximately equally profitable with the biggest difference being 10.7 dollars (Z=.28, p=.7795), which occurred in scenario 3 (N=10, C=20), accounting for only 5.07% of the expected revenue in scenario 3. As the theory predicts, in scenarios 5 and 6 where only asymmetry is present and N=2, selling two complements in a bundle is more profitable. When N=10, the two separate auction mechanisms generate approximately the same revenues, and both are more profitable than bundled auctions. A comparison of scenario 7 and 8 where N=10 shows that when C increases from 20 to 50, the revenue of auction for the bundle increases on average by $32.4(z=2.47, p=.0068, one tailed), the revenue of two sequential auctions increases only by $5.8 dollars (z=.27, p=.3936, one tailed), while the revenue of two simultaneous auctions actually decreases by $2.66 dollars (z=-.12, p=.4522, one tailed).

Table 4.6 Observed Revenues for the Eight Scenarios

Scenario		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number of Bidders		2	2	10	10	2	2	10	10
Complementarity		20	50	20	50	20	50	20	50
Heterogeneity	Variation	Yes	Yes	Yes	No	No	Yes	No	No
	Asymmetry	No	No	No	Yes	Yes	No	Yes	Yes
Bundled Auction		101.51 (52.73)	130.88 (56.52)	213.78 (23.85)	246.68 (26.11)	135.50 (12.67)	161.54 (20.68)	148.88 (8.02)	181.30 (10.37)
Simultaneous Auctions		105.80 (56.20)	128.66 (61.48)	224.54 (26.37)	252.19 (32.36)	109.20 (26.62)	142.92 (25.00)	214.24 (16.04)	211.58 (15.48)
Sequential Auctions		100.55 (56.12)	125.56 (58.20)	218.31 (27.15)	246.86 (29.88)	114.82 (26.34)	146.77 (27.36)	205.24 (16.26)	211.04 (14.08)

4.5 General Discussion

The primary research question in this study is "which of the three selling mechanisms is most profitable, when selling two complementarity products A and B?" Based on the theory and empirical evidence, we present the following findings.

None of the three mechanisms strictly dominates the others. Superiority in profitability of each of them depends on the heterogeneity of individual values, number of bidders and the magnitude of complementarity. The relative profitability of these three selling mechanisms depends on the net effect of the inefficiency in bundling and the exposure risk in separate auctions.

When there is high variation in product values, which are positively correlated, the three selling mechanisms are equally profitable, regardless of number of bidders and complementarity. This is due to both the absence of exposure risk in separate auctions and inefficiency in bundled auctions.

When high asymmetry exists and product values are negatively correlated, one bundled auction is more profitable than two separate auctions (simultaneous or sequential) when there are two bidders and less profitable when there are ten bidders. Simultaneous auctions and sequential auctions are approximately equally profitable, although the nature of exposure risk is different in these two separate selling strategies.

Appendix 4.1: Proof of Bayesian Nash Equilibria (BNE) in Two Simultaneous Separate Vickrey Auctions

Theorem 1 (Equilibrium Statement): *For two simultaneous separate auctions of N bidders (N=2, 10) in scenario 1, 2, 3 and 4, a BNE will consist of three type-contingent strategies.*

Types	**b_A**	**b_B**
T1	100+0.5C	100+0.5C
T3	60+0.5C	60+0.5C
T5	20+0.5C	20+0.5C

where C=20 in scenario 1 and 3, and 50 in scenario 2 and 4.

Proof: We need to prove that no bidder of any type has an incentive to depart from the BNE. First we examine T1 bidders. Each T1 bidder's two bids equal their corresponding values plus 0.5C. A T1 bidder has profit only when all other bidders are not T1. If there are multiply T1 bidders, say j, T1 bidders (2≤j≤N), each of them has 1/j of chance of winning both products with an expected profit of zero and (j-1)/j a chance of losing both products with an expected profit of zero. For a T1 bidder, the expected profit is

$$(\frac{2}{3})^{N-1} \cdot (V_{AT1} + V_{BT1} + C - b_A^{2nd} - b_B^{2nd})$$

where b_A^{2nd} and b_B^{2nd} is the expected second highest b_A and b_B when this bidder is the only T1 bidder.

A T1 bidder has no incentive to move from this equilibrium. Increasing her bid(s) won't increase expected profit. This is because when all her opponents are not T1, it doesn't increase her profit. When there are other T1 bidders, increasing one or both her bids lets her win the bundle, but her profit is always zero since the price she pays always equals the value of the bundle. Decreasing her bid(s) will not increase expected profit either. Because, again when all her opponents are not of T1, it does not increase her profit. And when there are other T1 bidders, decreasing her bid(s) lets her lose the bundle but her profit is still not changed.

Likewise, we can apply the same logic to prove other types of bidder will not depart from the BNE. For a T3 bidder, the sum of b_A and b_B is equal to the value of the bundle. Only when all of her opponents are of T5, a T3 bidder can have a profit. Increasing her bid(s) will not increase expected profit, because when some of her opponents are T1 and/or T3, she still either does not win any product or wins the bundle and pays a price equal to the value of the bundle. When her opponents are T5, increasing bid(s) does not increase profits. Decreasing her bid(s) will not increase expected profit, since when her opponents are T5, the result is not different.

For a T5 bidder, the sum of b_A and b_B is also equal to the value of the bundle. Only when all of her opponents are T5, has she a chance to win the bundle with zero profit. Increasing or decreasing one or both her bids does not increase profit.

Q.E.D

Theorem 2 (Equilibrium Statement): *For two simultaneous separate auctions of N bidders (N=2) in scenario 5, a BNE will consist of three type-contingent strategies.*

Types	b_A	b_B
T4	100	20+C
T3	60+0.5C	60+0.5C
T2	20+C	100

Where C=20 in scenario 5.

Proof: We need to prove that no bidder of any type has an incentive to depart from the BNE. First we examine T3 bidders. Each T3 bidder's two bids equal her corresponding values plus 0.5C. A T3 bidder has profit only when the other bidder is not T3. If there are two T3 bidders, each T1 bidder has 0.5 chance of winning both products with an expected profit of zero and 0.5 chance of losing both products with an expected profit of zero. For a T3 bidder, the expected profit is

$$(\frac{1}{3})\cdot(60-40)+(\frac{1}{3})\cdot(60-40)$$

A T3 bidder has no incentive to move from this equilibrium. Increasing her bid(s) will not increase expected profit. This is because when her opponent is not T3, it does not increase her profit. When her opponent is T3 bidders, increasing her bid(s) lets her win the bundle but her profit is always zero since the price she pays equals the value of the bundle. Decreasing her bid(s) will not increase expected profit either. Because, again when her opponent is not T3, it does not increase her profit. When her opponent is T3, decreasing her bid(s) lets her lose the bundle but her profit is zero.

Likewise, we can apply the same logic to prove other types of bidder will not depart from the BNE. For a T2 bidder, the sum of b_A and b_B is equal to the value of the bundle. Only when her opponent is T3 or T4, a T2 bidder can have a profit. A T2 bidder's expected profit is

$$(\frac{1}{3})\cdot(100-70)+(\frac{1}{3})\cdot(100-40)$$

Increasing her bid(s) will not increase her expected profit. Because when her opponent is T3 or T4, she still either does not win any product or wins the bundle and pays a price equal to the value of the bundle. When her opponent is T2, increasing bid(s) does not increase profits. Decreasing her bid(s) will not increase expected profit either.

The proof of the type contingent bids of T4 follows along the same lines.

Q.E.D

Theorem 3 (Equilibrium Statement): *For two simultaneous separate auctions of N bidders (N=2) in scenario 6, a BNE will consist of three type-contingent strategies.*

Types	b_{Ai} (i=1, 2)	b_{Bi} (i=1, 2)
T4	100	20+C
T3	100, i=1 70, i=2	100, i=1 70, i=2
T2	20+C	100

Where C=50 in scenario 6.

Proof: We need to prove that no bidder of any type has an incentive to depart from the BNE.

First we examine T3 bidders. Sum of bidder 1 (when she is T3)'s two bids is greater than the sum of two values plus C. Although she can always outbid her opponent, she has profit only when her opponent is T3. When her opponent is T2 or T4, bidder 1 can win both A and B and pay a total price equal to the value of the bundle. Therefore for bidder 1 (when she is T3), the expected profit is

$$(\frac{1}{3})\cdot(0)+(\frac{1}{3})\cdot 0+(\frac{1}{3})\cdot(30)$$

Bidder 1 has no incentive to move from this equilibrium because increasing her bid(s) won't increase expected profit. Nor does decreasing her bid(s) increase expected profit. Bidder 2's (when she is T3) expected profit is zero since she cannot win any product. Since bidder 1's (when she is T3) two bids put together are either equal to or greater than the value of the bundle, bidder 2 cannot increase her profit by increasing her bid(s). Similarly, decreasing bid(s) also does not increase bidder 2's profit.

The proof of the type contingent bids of T2 and T4 follows along the same lines as in Theorem 2.

Q.E.D

Theorem 4 (Equilibrium Statement): *For two simultaneous separate auctions of N bidders (N=10) in scenario 7, a BNE will consist of three type-contingent strategies.*

Types	b_A	b_B
T4	100	20+C
T3	70, i=1 60, i=2,...,10	70, i=1 60, i=2,...,10
T2	20+C	100

where C=20 in scenario 7.

Proof: We need to prove that no bidder of any type has an incentive to depart from the BNE.

First we examine T3 bidders. Sum of bidder 1's (when she is T3) two bids are equal to the corresponding values plus 0.5C. Bidder 1 has profit only when her opponents are all the same type (T2, T3 or T4). When her opponent is all T2 (T4), bidder 1 can win A (B) and have a profit of 20 dollars. When her opponent is all T3, bidder 1 can win both A and B and have a profit of 20 dollars. Therefore for bidder 1 (when she is T3), the expected profit is

$$(\frac{1}{3})^9 \cdot (20) + (\frac{1}{3})^9 \cdot (20) + (\frac{1}{3})^9 \cdot (20)$$

Bidder 1 has no incentive to move from this equilibrium. Increasing her bid(s) won't increase her expected profit because when all her opponents are T2 (T4), wining A (B) only is more profitable ($20) than winning both ($0). Decreasing her bid(s) does not increase her expected profit. Each of the other T3 bidders has a profit only when her opponents are either all T2 or all T4. When her opponents are all T2 (T4), this T3 bidder can win A (B) and have a profit of 20 dollars. Therefore, for one of other T3 bidders, the expected profit is

$$(\frac{1}{3})^9 \cdot (20) + (\frac{1}{3})^9 \cdot (20)$$

Each of other T3 bidders has no incentive to depart from this equilibrium. Increasing one or both her bids won't increase her expected profit. Nor does decreasing one or both her bids increase her expected profit.

The proof of the type contingent bids of T2 and T4 follows along the same lines as in Theorem 2.

Q.E.D

Theorem 5 (Equilibrium Statement): *For two simultaneous separate auctions of N bidders (N=10) in scenario 8, a BNE will consist of three type-contingent strategies.*

Types	**b_A**	**b_B**
T4	100	20+C
T3	70	70
T2	20+C	100

Where C=50 in scenario 8.

Proof: We need to prove that no bidder of any type has an incentive to depart from the BNE. First we examine T3 bidders. Each T3 Bidder 1 has a profit only when her opponents are all T3, and then she has 0.1 change of winning both products with a profit of 30 dollars. When some of her opponents are T2 or T4, a T3 bidder loses both products. Therefore for a T3 bidder, the expected profit is

$$(\frac{1}{3})^9 \cdot (\frac{20}{10})$$

A T3 bidder has no incentive to move from this equilibrium. Decreasing her bid(s) won't increase her expected profit because she will not win any product and have a profit. Suppose a T3 bidder adds a small amount θ to her b_A, her profit will be

$$(\frac{1}{3})^9 \cdot (20) + \left[(\frac{2}{3})^9 - (\frac{1}{3})^9\right] \cdot (-10) < (\frac{1}{3})^9 \cdot (\frac{20}{9})$$

This is because when there are both T2 and T3 bidders, she will lose 10 dollars. So increasing her bids will not increase her expected profit. Decreasing her bid(s) will not let her have a chance of winning the bundle with profit and decrease her expect profit to zero.

The proof of the type contingent bids of T2 and T4 follows along the same lines as in Theorem 2.

Q.E.D

Appendix 4.2: Proof of Bayesian Nash Equilibria (BNE) in two Sequential Separate Vickrey Auctions

Theorem 1 (Equilibrium Statement): *For two sequential separate auctions of N bidders (N=2, 10) in scenario 1, 2, 3 and 4, a BNE will consist of three type-contingent strategies.*

Types	b_A	$b_{B\|winA}$	$b_{B\|loseA}$
T1	100+C	100+C	100
T3	60+C	60+C	60
T5	20+C	20+C	20

where C=20 in scenario 1 and 3 and 50 in scenario 2 and 4.

Proof: 1) The type contingent bids in the second auction for product B

In the second auction, if a bidder wins the first auction for A, the marginal profit of winning B is (V_B+C-2nd highest bid on B). So the optimal strategy in the second auction, conditional on winning A, is to bid (V_B+C). If a bidder loses the first auction for A, the marginal profit of winning B is (V_B-2nd highest bid on B) and therefore the optimal strategy in the second auction, conditional on losing A, is to bid V_B.

2) The type contingent bids in the first auction for product A

In the first auction, for T1 bidders, the type contingent bid on A is (100+C). For each of the T1 bidders, the expected profit in the two auctions will be

$$\sum_{n1=1}^{n-1}\frac{(n-1)!}{(n-1-n1)!n1!}\left(\frac{1}{3}\right)^{n-1}\frac{(-C+C)}{n1+1}+R_{35}=R_{35}$$

where n1 is the number of opponents of type T1, and P_{35} is the expected revenue when all opponents are of T3 and T5. So when there are more than (n1+1) T1 bidders, each T1 bidder has an expected profit of zero no matter whether she wins (with 1/(n1+1) chance and pays a price equal to 100+C) or not. A T1 bidder can have positive profit only when all her opponents are of T3 and T5.

Let us suppose one of the T1 bidders increases her bid on A to be (100+C+θ), where θ is a small amount. Now this bidder can win B when there are other T1 bidders. The expected profit would be

$$\sum_{n1=1}^{n-1}\frac{(n-1)!}{(n-1-n1)!n1!}\left(\frac{1}{3}\right)^{n-1}(-C+C)+R_{35}=R_{35}$$

This is the same as before because the price she pays equals 100+C when there are other T1 bidders. Similarly if she decreases her bid on A to be (100+C-θ), where θ is a small amount, the expected profit will still be the same. Therefore, there is no incentive to depart from the BNE.

For T3 bidders, the type contingent bid on A is (60+C). Then, for each of the T1 bidders, the expected profit in the two auctions will be

$$\sum_{n3=1}^{n-1}\frac{(n-1)!}{(n-1-n3)!n3!}\left(\frac{1}{3}\right)^{n-1}\frac{(-C+C)}{n3+1}+R_{5}=R_{5}$$

where n3 is the number of opponents of type T3, and R_5 is the expected revenue when all opponents are of T5. So when there is at least one T1 bidder and /or more then two T3 bidders, each T3 bidder has an expected profit of zero. A T3 bidder can have only positive profit when all her opponents are of type T5.

Let us suppose one of the T3 bidder increases her bid on A to be (60+C+θ), where θ is a small amount. The expect profit would be

$$\sum_{n3=1}^{n-1} \frac{(n-1)!}{(n-1-n3)!\,n3!}\left(\frac{1}{3}\right)^{n-1}(-C+C)+P_5 = P_5$$

It is the same as the previous profit. If she decreases her bid on A to be (60+C-θ), where θ is a small amount, the expected profit would be P_5. It is still the same as the previous profit. So there is no incentive to depart from the BNE.

The proof of the type contingent bids of T5 follows along the same lines.

Q.E.D

Theorem 2 (Equilibrium Statement): *For two sequential separate auctions of two bidders in scenario 5 and 6, a BNE will consist of three type-contingent strategies.*

Types	b_A	$b_{B\|winA}$	$b_{B\|loseA}$
T4	100+C	20+C	20
T3	60+C	60+C	60
T2	20+C	100+C	100

where C=20 in scenario 5, and 50 in scenario 6.

Proof: (1) Scenario 5

When A is won in the first auction, for each bidder the optimal strategy in the second auction is to bid (V_B+20). If A is not won, the optimal strategy in the second auction is to bid V_B. For proof please see theorem 1.

In the first auction, each bidder's type contingent bid on A equals (V_A+20).

For a T4 bidder, when her opponent is of T4, a T4 bidder has an expected profit of zero. A T4 bidder can have positive profit only when her only opponent is of T2 or T3. Her expected profit is as follows,

$$\left(\frac{1}{3}\right)\cdot 60+\left(\frac{1}{3}\right)\cdot 20+\left(\frac{1}{3}\right)\cdot 0$$

Let us suppose one of the T4 bidders increases her bid on A to be (100+20+θ), where θ is a small amount. The expected profit would be the same as the previous profit. And if she decreases her bid on A to be (100+20-θ), where θ is a small amount, the expected profit would be zero. So there is no incentive to depart from the BNE.

For T2 and T3 bidders, the proofs follow along the same lines.

(2) Scenario 6

When A is won in the first auction, for each bidder the optimal strategy in the second auction is to bid (V_B+50). If she loses the first auction for A, the optimal strategy in the second auction is to bid V_B. For proof please see the theorem 1.

In the first auction, type contingent bids on A are (VA+50).

For a T4 bidder, when her opponent is of T4 or T3, a T4 bidder has an expected profit of zero. A T4 bidder can have positive profit only when her opponent is of T2. Her expected profit is as follows:

$$\left(\frac{1}{3}\right)\frac{(-50+50)}{2}+\left(\frac{1}{3}\right)\cdot(-10+10)+\left(\frac{1}{3}\right)\cdot(100-70)=10$$

Let us suppose one of the T4 bidders increases her bid on A to be (100+20+θ), where θ is a small amount. The expected profit would be the same as the previous profit. And if she decreases her bid on A to be (100+20-θ), where θ is a small amount, the expected profit would be still the same as the previous profit. So there is no incentive to depart from the BNE.

The proof of the type contingent bids of T3 and T2 follows along the same lines.

Q.E.D

Theorem 3 (Equilibrium Statement): *For two sequential separate auctions of ten bidders in scenario 7, a BNE will consist of three type-contingent strategies.*

Types	b_A	$b_{B\|winA}$	$b_{B\|loseA}$
T4	100+x4	40	20
T3	60+x3	80	60
T2	40	120	100

where $0 \leq x4 \leq 20$, $0 \leq x3 \leq 20$. x3 and x4 satisfy continuous distribution whose cumulative distribution functions will be defined later in the proof.

Proof: When A is won, for each bidder the optimal strategy in the second auction is to bid (V_B+20). If she loses the first auction for A, the optimal strategy in the second auction is to bid V_B. For proof please see Theorem 1.

In the first auction, suppose T2, T3 and T4's type contingent bids on A are respectively 40, (60+x3), and (100+x4), where $40 < (60+x3) < (100+x4)$.

Each T2 bidder has the following expected profit

$$\sum_{n2=1}^{n-1} \frac{(n-1)!}{(n-1-n2)!\,n2!} \left(\frac{1}{3}\right)^{n-1} \frac{(-C+C)}{n2+1} = 0$$

where n2 is the number of her opponents of type T2. Increasing the bid on A to (40+θ) will result in a profit of,

$$\sum_{n2=1}^{n-1} \frac{(n-1)!}{(n-1-n2)!\,n2!} \left(\frac{1}{3}\right)^{n-1} (-C+C) = 0$$

Decreasing the bid on A to (40-θ) will still make the profit zero.

For T3 bidders, the type contingent bid is (60+x3) on A, where $0 \leq x3 \leq 20$, and x3 satisfies a continuous distribution with an accumulative distribution function $F_3(x)$. By bidding (60+x3) on A, each T3 bidder's expected profit consists of the following components;

(a) When all her opponents are T2

$$\left(\frac{1}{3}\right)^{n-1} (60-40)$$

(b) When all her opponents are T3

$$\left(\frac{1}{3}\right)^{n-1}(20-x3)\cdot F_3^{n-1}(x3)$$

(c) When all her opponents are T4

$$\left(\frac{1}{3}\right)^{n-1}(60-40)$$

(d) When n2 opponents are T2 and n3 opponents are T3, where n2+n3=n-1

$$\sum_{n3=1}^{n-2}\frac{(n-1)!}{(n-1-n3)!\cdot n3!}\left(\frac{1}{3}\right)^{n-1}(-x3)F_3^{n3}(x3)$$

At BNE, the profit of bidding any 60+x3 (0≤x3≤20) should be equal to the profit of bidding 60, which is the sum of (a) and (c). Therefore we have the following equation,

$$\left(\frac{1}{3}\right)^{n-1}(20-x3)\cdot F_3^{n-1}(x3)+\sum_{n3=1}^{n-2}\frac{(n-1)!}{(n-1-n3)!\cdot n3!}\left(\frac{1}{3}\right)^{n-1}(-x3)F_3^{n3}(x3)=0$$

Since there is not a closed form solution for $F_3(x3)$, we use numerical simulation to estimate this equation. The simulation shows when $F_3(x3) = 0.25$, $x3=1.18\times10^{-5}$, $F_3(x3) = 0.5$, $x3=1.04\times10^{-3}$, $F_3(x3) = 0.8$, $x3=0.014$, $F_3(x3) = 0.975$, $x3=0.039$. Therefore, the expected value of x3 is 0.006889, and, in general, T3 bidder's type contingent bid on A is very close to 60. For the sake of simplicity, we use 60 as an estimate for T3 bidder's bid on A in the following proof and for calculating the revenue.

For T4 bidders, the type contingent bid is (100+x4) for A, where 0≤ x4 ≤20, and x4 satisfies a continuous distribution with the accumulative distribution function of $F_4(x)$. By bidding (100+x4) on A, each T4 bidder's expected profit consists of the following components,

(a) When all her opponents are T2

$$\left(\frac{1}{3}\right)^{n-1}(100-40)$$

(b) When all her opponents are T3s

$$\left(\frac{1}{3}\right)^{n-1}(100-60)$$

(c) When all her opponents are T4s

$$\left(\frac{1}{3}\right)^{n-1}(20-x4)\cdot F_4^{n-1}(x4)$$

(d) When n23 opponents are T2s or T3s, n4 opponents are T4, where n23+n4=n-1

$$\sum_{n4=1}^{n-2}\frac{(n-1)!}{(n-1-n4)!\cdot n4!}\left(\frac{1}{3}\right)^{n4}\left(\frac{2}{3}\right)^{n-1-n4}(-x4)F_4^{n4}(x4)$$

At BNE, the profit of bidding any 60+x3 ($0\leq x3\leq 20$) should be equal to the profit of bidding 60, which is the sum of (a) and (c). Therefore we have the following equation,

$$\left(\frac{1}{3}\right)^{n-1}(20-x4)\cdot F_4^{n-1}(x4)+\sum_{n4=1}^{n-2}\frac{(n-1)!}{(n-1-n4)!\cdot n4!}\left(\frac{1}{3}\right)^{n4}\left(\frac{2}{3}\right)^{n-1-n4}(-x4)F_4^{n4}(x4)=0$$

Again, since there is not a closed form solution for $F_4(x3)$, we use numerical simulation to estimate this equation. The simulation shows when $F_4(x4) = 0.25$, $x4=7.899\times10^{-8}$, $F_4(x4) = 0.5$, $x4=1.183\times10^{-5}$, $F_4(x4) = 0.75$, $x4=1.77\times10^{-4}$, $F_4(x4) = 0.975$, $x4=8.795\times10^{-4}$. The expected value of x 4 is 0.00015. Generally, T4 bidder's type contingent bid on A is very close to 100. For the sake of simplicity, we use 100 as estimation for T3 bidder's bid on A in calculating the revenue.

Q.E.D

Theorem 4 (Equilibrium Statement): *For two sequential separate auctions of ten bidders in scenario 8, a BNE will consist of three type-contingent strategies.*

Types	b_A	$b_{B\|winA}$	$b_{B\|loseA}$
T4	100+x4'	70	20
T3	60+x3'	110	60
T2	70	150	100

where 0≤x4'≤20, 0≤x3'≤20. x3'and x4' satisfy continuous distribution whose cumulative distribution functions will be defined later in the proof.

Proof: When A is won in the first auction, for each bidder the optimal strategy in the second auction is to bid (V_B+50). If she loses the first auction for A, the optimal strategy in the second auction is to bid V_B. For proof please see the theorem 1.

In the first auction, T2, T3 and T4's type contingent bids on A are respectively 70, (60+x3'), and (100+x4'), where 70 < (60+x3') < (100+x4').

Each T2 bidder has the following expected profit

$$\sum_{n2=1}^{n-1} \frac{(n-1)!}{(n-1-n2)!\,n2!}\left(\frac{1}{3}\right)^{n-1} \frac{(-50+50)}{n2+1} = 0$$

where n2 is the number of her opponents to be T2. Increasing the bid on A to be (70+θ) will make the following profit

$$\sum_{n2=1}^{n-1} \frac{(n-1)!}{(n-1-n2)!\,n2!}\left(\frac{1}{3}\right)^{n-1} (-C+C) = 0$$

Decreasing the bid on A to be (70-θ) will make the profit zero.

For T3 bidders, the type contingent bid is (60+x3') on A, where $10 \le x3' \le 50$, and x3' satisfies a continuous distribution with the accumulative distribution function of $F_{3'}(x)$. By bidding (60+x3') on A, each T3 bidder's expected profit consists of the following components:

(a) when all her opponents are T3

$$\left(\frac{1}{3}\right)^{n-1}(20-x3)\cdot F_{3'}^{n-1}(x3')$$

(b) n2 opponents are T2 and n3 opponents are T3, where n2+n3=n-1

$$\sum_{n3=1}^{n-2}\frac{(n-1)!}{(n-1-n3)!\cdot n3!}\left(\frac{1}{3}\right)^{n-1}(10-x3)F_{3'}^{n3}(x3')$$

At BNE, the profit of bidding any 60+x3' ($10 \le x3' \le 50$) should be equal to the profit of bidding 60+10=70, which is zero. Therefore we have the following equation

$$\left(\frac{1}{3}\right)^{n-1}(20-x3)\cdot F_{3'}^{n-1}(x3')+\sum_{n3=1}^{n-2}\frac{(n-1)!}{(n-1-n3)!\cdot n3!}\left(\frac{1}{3}\right)^{n-1}(10-x3)F_{3'}^{n3}(x3')=0$$

Since there is not a closed form solution for $F_{3'}(x)$, we use numerical simulation to estimate this equation. The simulation shows when $F_{3'}(x3') = 0.25$, x3'=10, $F_{3'}(x3') = 0.5$, x3'=10.001, $F_{3'}(x3') = 0.75$, x3'=10.005, $F_{3'}(x3') = 0.975$, x3'=10.017. The expected value of x3' is 10.003. Generally, T3 bidder's type contingent bid on A is very close to 70. For the sake of simplicity, we use 70 as an estimation for T3 bidder's bid on A in the following proof and for calculating the revenue.

For T4 bidders, the type contingent bid is (100+x4') on A, where $0 \le x4' \le 50$, and x4'satisfies a continuous distribution with the accumulative distribution function of $F_{4'}(x)$.

By bidding (100+x4') on A, each T4 bidder's expected profit consists of the following components,

(a) when all her opponents are T2,

$$\left(\frac{1}{3}\right)^{n-1}(100-70)$$

(b) when all her opponents are T3,

$$\left(\frac{1}{3}\right)^{n-1}(50-x3')$$

(c) when all her opponents are T4,

$$\left(\frac{1}{3}\right)^{n-1}(50-x4')\cdot F_{4'}^{n-1}(x4')$$

(d) when n3 opponents are T3 and n4 opponents are T4, where n3+n4=n-1, n3>0 and n4>0

$$\sum_{n4=1}^{n-2}\frac{(n-1)!}{(n-1-n4)!\cdot n4!}\left(\frac{1}{3}\right)^{n-1}(70-60-x4')F_{4'}^{n4}(x4')$$

(e) when n2 opponents are T2, n3 opponents are T3 and n4 opponents are T4, where n2+n3+n4=n-1, n2, n3, n4>0

$$\sum_{n4=1}^{n-2}\sum_{n3=1}^{n-2-n4}\frac{(n-1)!}{(n-1-n4-n3)!\cdot n4!\cdot n3!}\left(\frac{1}{3}\right)^{n-1}(-x4)F_{4'}^{n4}(x4')$$

At BNE, the profit of bidding any 100+x4' (0≤x4'≤50) on A should be equal to the profit of bidding 100, which is the sum of (a) and (b). Therefore we have the following equation,

(c)+(d)+(e)=0

Since there is not a closed form solution for $F_{4'}(x)$, we use numerical simulation to estimate this equation. The simulation shows when $F_{4'}(x4') = 0.25$, $x4' = 0.067$, $F_{4'}(x4') = 0.5$, $x4' = 0.115$, $F_{4'}(x4') = 0.75$, $x4' = 0.184$, $F_{4'}(x4') = 0.975$, $x4' = 0.266$. The expected value of $x4'$ is 0.13. Generally, T3 bidder's type contingent bid on A is very close to 100. For the sake of simplicity, we use 100.13 as estimation for T3 bidder's bid on A in calculating the revenue.

Q.E.D

Appendix 4.3: Revenue of One Bundled Auction

(1) Scenario 1 to 4

Figure 4.2 demonstrates all potential revenues in one bundled auction. Table 4.7(A) shows the amounts of all the potential revenues in Scenario 1, 2, 3 and 4. Table 4.7(B) shows the likelihoods of all the potential revenues in Scenario 1 and 3 and Table 4.7(C) shows Scenario 2 and 4. The expected revenue in a given scenario is equal to the weighted sum of all three potential revenues.

Figure 4.2 Revenues in Scenario 1 to 4

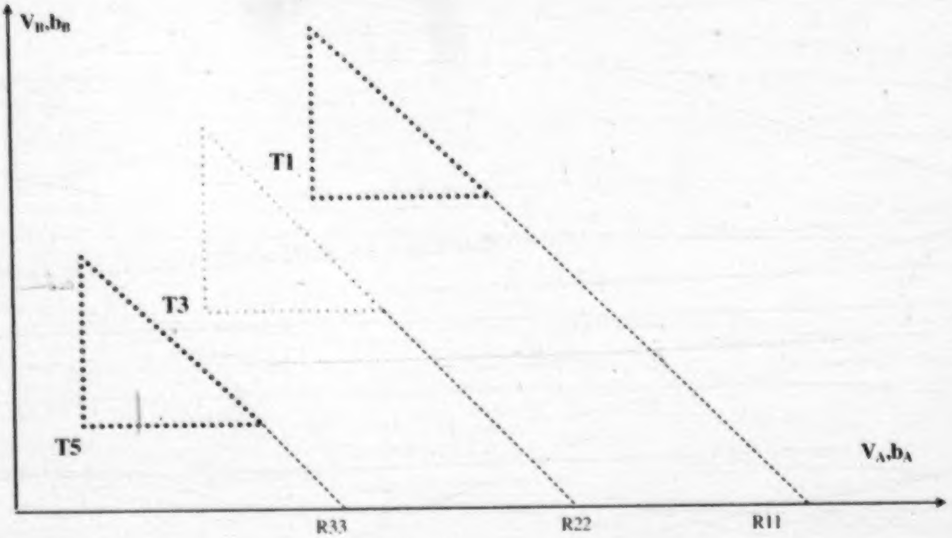


Table 4.7(A) Potential Revenues of One Bundled Auction in Scenario 1, 2, 3 and 4

Revenues	Value
R11	200+C
R22	120+C
R33	40+C

Table 4.7(B) Potential Revenues of One Bundled Auction and Likelihoods in Scenario 1 and 3

Revenue	Likelihood	Explanation
R11	0.3333×0.3333	Two (T1)
R22	0.3333×0.3333×2 +0.3333×0.3333	One(T3), ONE(T1) Or Two (T3)
R33	2×0.3333×0.6666 +0.3333×0.3333	One(T5), One(T1,T3) Or Two (T5)

Table 4.7C) Potential Revenues of One Bundled Auction and Likelihoods in Scenario 2 and 4

Revenue	Likelihood	Explanation
R11	$[1-0.6666^{N} - N\times0.3333\times0.6666^{(N-1)}]$	Two (T1), Rest (all)
R22	$[0.6666^{N}-0.3333^{N} - N\times0.3333\times0.3333^{(N-1)}]$ $+N\times0.3333\times[0.6666^{N-1}-0.3333^{N-1}]$	Two (T3), Rest (T5, T3) Or One (T1), One(T3), Rest (T5,T3)
R33	$N\times0.6666\times0.3333^{N-1}$ $+ 0.3333^{N}$	One(T1,T3), Rest(T5), Or All(T5)

(2) Scenario 5 to 8

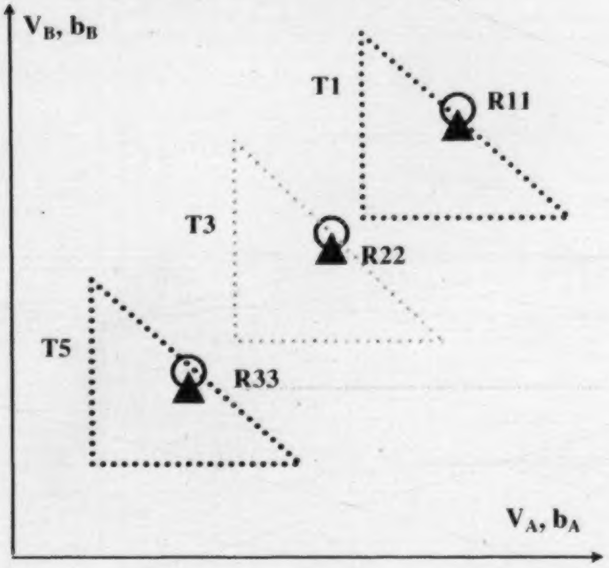
In these four scenarios all bidders have same value of bundle, (120+C), which is the revenue of the bundled auction.

Appendix 4.4: Revenue of Two Simultaneous Auctions

1) Scenario 1 to 4

In these four scenarios, the magnitudes of three potential revenues and likelihoods of these revenues are the same as in bundled auction in scenario 1 to 4. See Appendix 3 for detail.

Figure 4.3 Revenues in Scenario 1 to 4



2) Scenario 5

Based on Appendix 1 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.4 demonstrates all these potential revenues (the small triangles are potential revenues and small circles are type contingent strategies). Table 4.8(A) shows the amounts of all the potential revenues in Scenario 5 and Table 4.8(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.4 Revenues in Scenario 5

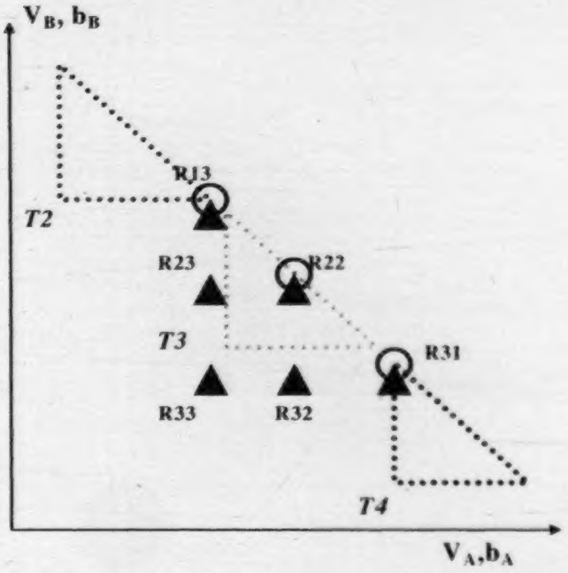

V_B, b_B
R13
T2
R23
R22
T3
R31
R33
R32
T4
V_A, b_A

Table 4.8(A) Potential Revenues of Simultaneous Auctions in Scenario 5

Revenue	Amount
R33	80
R23, R32	110
R13, R31	140
R22	140
R12, R21	170
R11	200

Table 4.8(B) Potential Revenues of Simultaneous Auctions and Likelihoods in Scenario 5

Revenue	Likelihood	Explanation
R33	2 0.3333×0.3333	One (T2), One (T4)
R23, R32	2×0.3333×0.3333×2	For R23: One (T2), One (T3)
R13, R31	0.3333×0.3333×2	For R13: Two (T2)
R22	0.3333×0.3333	Two (T3)

3) Scenario 6

Based on Appendix 1 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.5 demonstrates all these potential revenues (the small triangles are potential revenues and small circles are type contingent strategies). Table 4.9(A) shows the amounts of all the potential revenues in Scenario 6, and Table 4.9(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.5 Revenues in Scenario 6

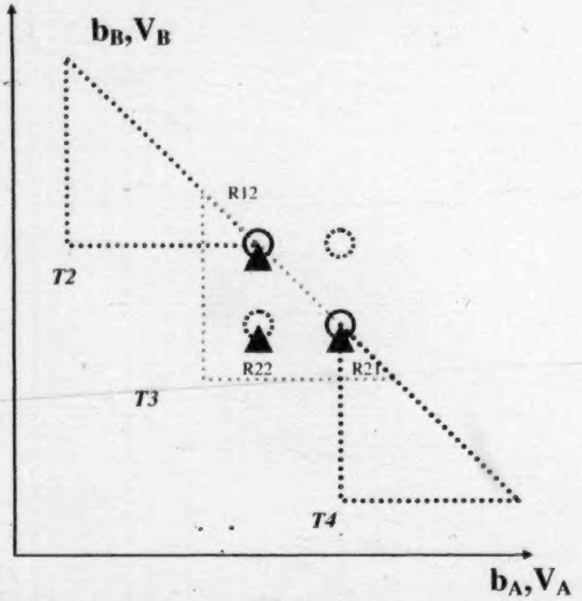


Table 4.9(A) Potential Revenues of Simultaneous Auctions in Scenario 6

Revenue	Amount
R22	140
R12	170
R21	170

Table 4.9(B) Potential Revenues of Simultaneous Auctions and Likelihoods in Scenario 6

Revenue	Likelihood	Explanation
R22	0.3333×0.3333 +0.3333 +0.3333×0.3333	Bidder2 (T2), bidder 1 (T4) Or bidder 2 (T3), bidder 1 (T2, T3, T4) Or bidder 2 (T4), bidder 1 (T2)
R12, R21	0.3333×0.6666×2	For R12: Bidder 2 (T2), bidder 1(T2,T3)

4) Scenario 7

Based on Appendix 1 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.6 demonstrates all these potential revenues (the small triangles are potential revenues and small circles are type contingent strategies). Table 4.10(A) shows the amounts of all the potential revenues in Scenario 7, and Table 4.10(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.6 Revenues in Scenario 7

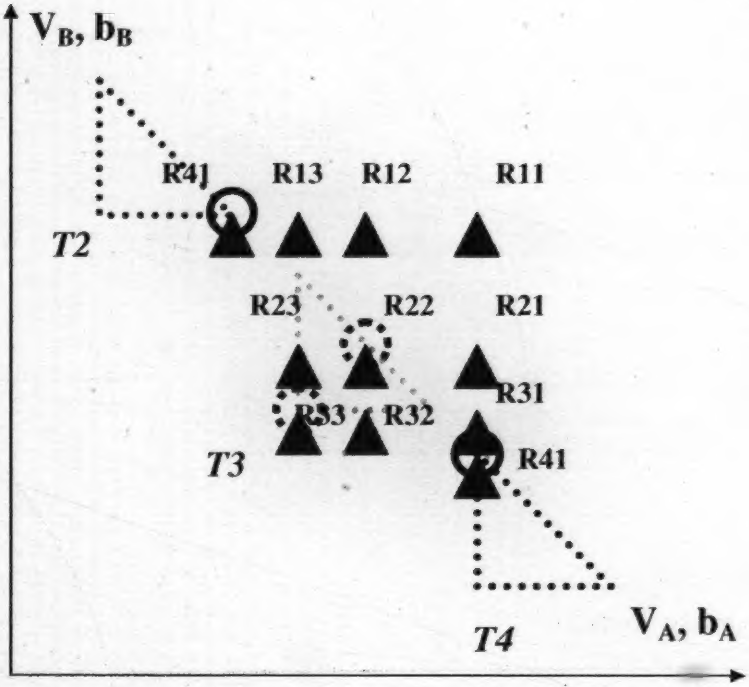


Table 4.10(A) Potential Revenues of Simultaneous Auctions in Scenario 7

Revenue	Amount
R11	200
R12, R21	170
R13, R31	160
R14, R41	140
R22	140
R23, R32	130
R33	120

Table 4.10(B) Potential Revenues of Simultaneous Auctions and Likelihoods in Scenario 7

Revenue	Likelihood	Explanation
R11	$[1-0.6666^{N}-0.6666^{N}-N\times0.3333\times0.6666^{(N-1)}-N\times0.3333\times0.6666^{(N-1)}+0.3333^{N}+N\times0.3333\times0.3333^{(N-1)}+N\times0.3333\times0.3333^{(N-1)}+N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}]$	Two(T2), Two(T4), Rest (all)
R12, R21	$\{0.3333\times(N-1)\times0.3333\times[0.6666^{(N-2)}-0.3333^{(N-2)}-(N-2)\times0.3333\times0.3333^{(N-3)}]\}\times2$	R12: B1(T3), One(T4), Two(T2), Rest (T2,T3)

Table 4.10(B) Potential Revenues of Simultaneous Auctions and Likelihoods in Scenario 7 (continued)

Revenue	Likelihood	Explanation
R13, R31	$\{0.3333\times(0.6666^{(N-1)}-0.3333^{(N-1)}-0.3333^{(N-1)}-(N-1)\times0.3333\times0.3333^{(N-2)})$ $+0.3333\times(N-1)\times0.3333\times[0.6666^{(N-2)}-0.3333^{(N-2)}-0.3333^{(N-2)}]$ $+0.3333\times(0.6666^{(N-1)}-0.3333^{(N-1)}-0.3333^{(N-1)}-(N-1)\times0.3333\times0.3333^{(N-2)})$ $+0.3333\times(0.6666^{(N-1)}-0.3333^{(N-1)}-0.3333^{(N-1)}-(N-1)\times0.3333\times0.3333^{(N-2)})$ $\}\times2$	R13: B1(T2), One(T2), Two(T3), Rest(T2,T3); Or B1(T2), One(T2),One(T3), One(T4), Rest(T2,T3); Or B1(T3), Two(T2), One(T3),Rest(T2,T3); Or B1(T4), Two(T2), one(T3), rest(T2, T3);
R14, R41	$\{0.3333\times(N-1)\times0.6666\times0.3333^{(N-2)}$ $+0.3333\times0.3333^{(N-1)}$ $+0.6666\times0.3333^{(N-1)}$ $\}\times2$	R14: B1(T2), One(T3,T4), Rest(T2); Or B1(T2), Rest(T2); Or B1(T3, T4), Rest(T2);
R22	$0.3333\times(N-1)\times0.3333\times(N-2)\times0.3333\times0.3333^{(N-3)}$	B1(T3), One(T2), One(T4), Rest(T3);
R23,R32	$\{0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}$ $\}\times2$	R23: B1(T3), One(T2), Rest(T3);
R33	$0.3333\times0.3333^{(N-1)}$ $+0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}$ $+0.3333\times0.3333^{(N-1)}$ $+0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}$ $+0.3333\times0.3333^{(N-1)}$	B1(T3), Rest(T3); Or B1(T2), One(T4), Rest(T3); Or B1(T2), Rest(T3); Or B1(T4), One(T4), Rest(T3); Or B1(T4), Rest(T3);

5) Scenario 8

Based on Appendix 1 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.7 demonstrates all these potential revenues (the small triangles are potential revenues and small circles are type contingent strategies). Table 4.11(A) shows the amounts of all the potential revenues in Scenario 8, and Table 4.11(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.7 Revenues in Scenario 8

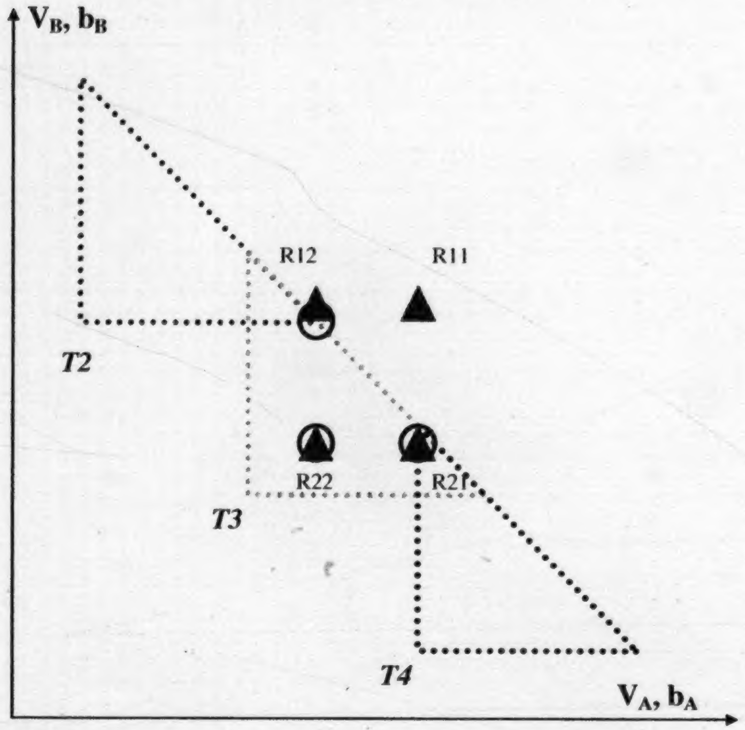


Table 4.11(A) Potential Revenues of Simultaneous Auctions in Scenario 8

Revenue	Amount
R22	140
R12	170
R21	170

Table 4.11(B) Potential Revenues of Simultaneous Auctions and Likelihoods in Scenario 8

Revenue	Likelihood	Explanation
R22	$N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}$ $+ N\times0.6666\times0.3333^{(N-1)}$ $+ 0.3333^{N}$	One (T2), One (T4), Rest (T3) Or One (T2, T4), Rest (T3), Or All (T3)
R12	$N\times0.3333\times[0.6666^{(N-1)} -0.3333^{(N-1)}-(N-1)\times0.3333\times 0.3333^{(N-2)}]$ $+[0.6666^{N} -0.3333^{N}-N\times0.3333\times 0.3333^{(N-1)}]$	Two (T2), One (T4), Rest (T2,T3) Or Two (T2), Rest (T2,T3)
R21	$N\times0.3333\times[0.6666^{(N-1)} -0.3333^{(N-1)}-(N-1)\times0.3333\times 0.3333^{(N-2)}]$ $+[0.6666^{N} -0.3333^{N}-N\times0.3333\times 0.3333^{(N-1)}]$	Two (T4), One (T2) Rest (T4,T3) Or Two (T4), Rest (T4,T3)
R11	$[1-0.6666^{N}-0.6666^{N}-N\times0.3333\times0.6666^{(N-1)} -N\times0.3333\times0.6666^{(N-1)}+0.3333^{N}+N\times0.3333\times0.3333^{(N-1)} +N\times0.3333\times0.3333^{(N-1)} +N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}]$	Two (T4),Two (T2), Rest (all)

Appendix 4.5: Revenue of Two Sequential Separate Auctions

1) Scenario 1 to 4

Based on Appendix 2 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.8 demonstrates all these potential revenues (the small triangles are potential revenues). Table 4.12(A) shows the amounts of all the potential revenues in Scenario 1 to 4, and Table 4.12(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.8 Revenues in Scenario 1 to 4

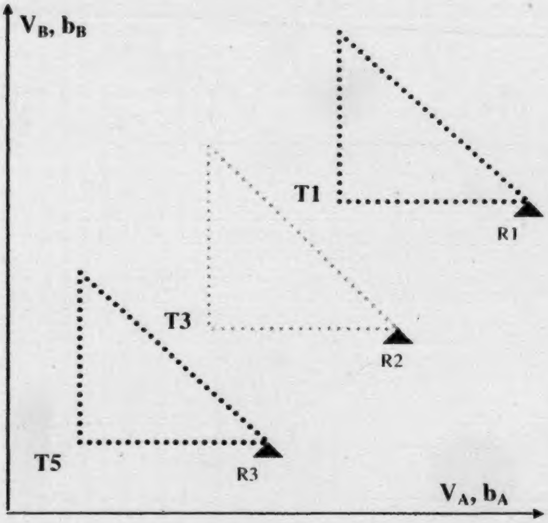


Table 4.12(A) Potential Revenues of Sequential Auctions in Scenario 1 to 4

Revenue	Amount
R1	200+C
R2	120+C
R3	40+C

Table 4.12(B) Potential Revenues of Sequential Auctions and Likelihoods in Scenario 1 to 4

Revenue	Likelihood	Explanation
R1	$[1-0.6666^{N} - N\times 0.3333\times 0.6666^{(N-1)}]$	Two (T1), Rest (all)
R2	$[0.6666^{N}-0.3333^{N} - N\times 0.3333\times 0.3333^{(N-1)}]$ $+N\times 0.3333\times[0.6666^{N-1}-0.3333^{N-1}]$	Two (T3), Rest (T3, T5) Or One(T1), One(T3), Rest(T3,T5)
R3	$N\times 0.6666\times 0.3333^{N-1}$ $+ 0.3333^{N}$	One(T1,T3), Rest(T5), All(T5)

2) Scenario 5

Based on Appendix 2 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.9 demonstrates all these potential revenues (the small triangles are potential revenues). Table 4.13(A) shows the amounts of all the potential revenues in Scenario 5, and Table 4.13(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Table 4.13(A) Potential Revenues of Sequential Auctions in Scenario 5

Revenue	Amount
R44	140
R33	140
R22	140
R43	120
R42	80
R32	120

Figure 4.9 Revenues in Scenario 5

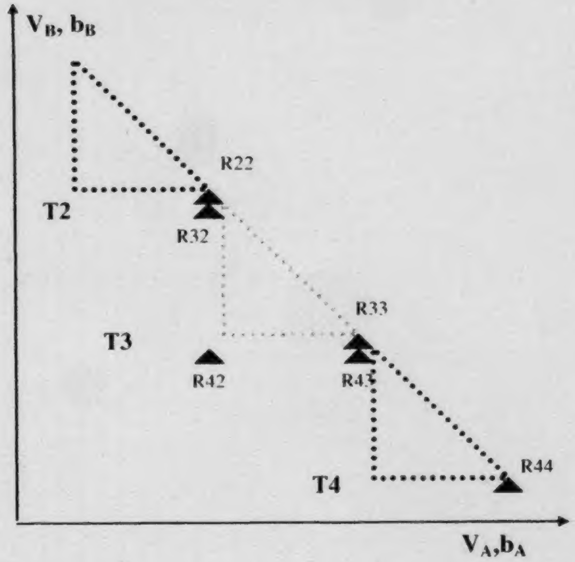


Table 4.13(B) Potential Revenues of Sequential Auctions and Likelihoods in Scenario 5

Revenue	Likelihood	Explanation
R44	0.3333^2	Two (T4)
R33	0.3333^2	Two (T3)
R22	0.3333^2	Two (T2)
R43	2×0.3333^2	One(T4), One(T3)
R42	2×0.3333^2	One(T4), One(T2)
R32	2×0.3333^2	One(T3), One(T2)

3) Scenario 6

Based on Appendix 2 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.10 demonstrates all these potential revenues (the small triangles are potential revenues). Table 4.14(A) shows the amounts of all the potential revenues in Scenario 6, and Table 4.14(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.10 Revenues in Scenario 6

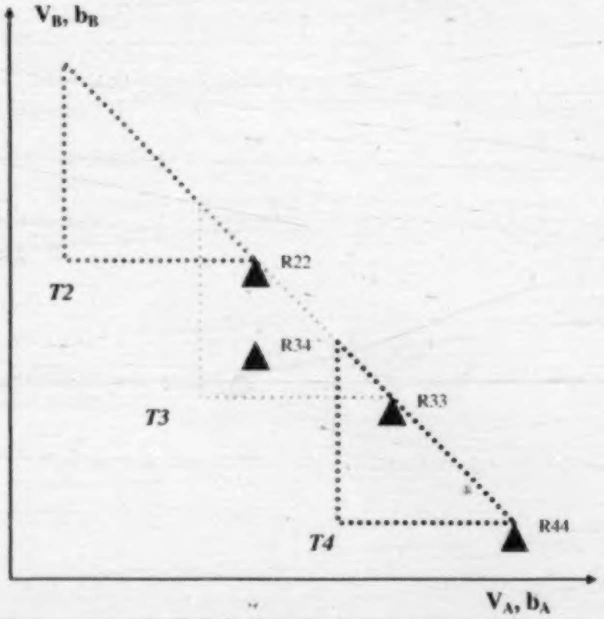


Table 4.14(A) Potential Revenues of Sequential Auctions in Scenario 6

Revenue	Amount
R44	170
R33	170
R22	170
R34	140

Table 4.14(B) Potential Revenues of Sequential Auctions and Likelihoods in Scenario 6

Revenue	Likelihood	Explanation
R44	0.3333^2	Two (T4)
R33	0.3333^2 $+2\times0.3333^2$	Two (T3) One(T4), One(T3)
R22	0.3333^2 $+2\times0.3333^2$	Two (T2) One(T2), One(T3)
R34	2×0.3333^2	One(T3), One(T4)

4) Scenario 7

Based on Appendix 2 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.11 demonstrates all these potential revenues (the small triangles are potential revenues). Table 4.15(A) shows the amounts of all the potential revenues in Scenario 7, and Table 4.15(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

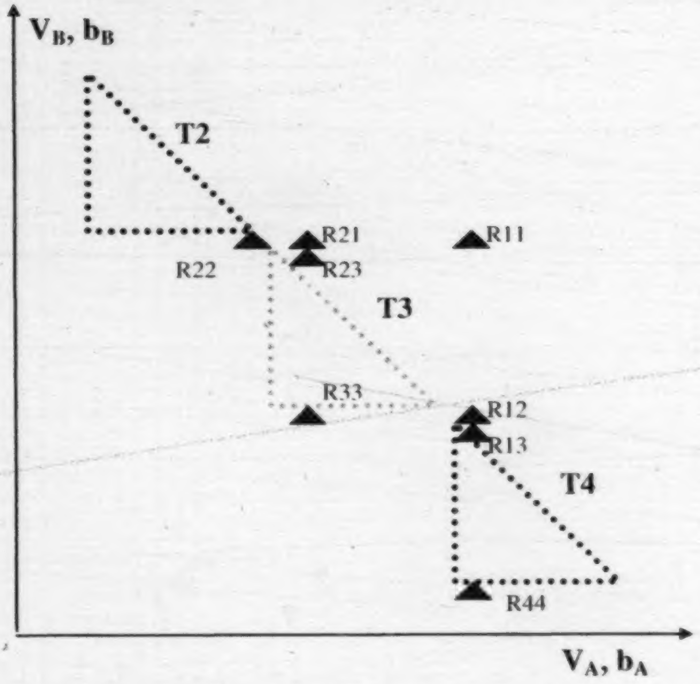
Table 4.15(A) Potential Revenues of Sequential Auctions in Scenario 7

Revenue	Amount
R44	120
R33	120
R22	140
R11	200
R12	160
R21	160

Table 4.15(A) Potential Revenues of Sequential Auctions in Scenario 7 (continued)

Revenue	Amount
R13	140
R23	140

Figure 4.11 Revenues in Scenario 7



V_B, b_B
T2
R21
R11
R22
R23
T3
R33
R12
R13
T4
R44
V_A, b_A

Table 4.15(B) Potential Revenues of Sequential Auctions and Likelihoods in Scenario 7

Revenue	Likelihood	Explanation
R44	0.3333^{N}	All (T4);
R33	0.3333^{N}	All (T3);
	$+N\times0.3333\times0.3333^{(N-1)}$	Or One (T4), Rest (T3);
	$+ N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}$	Or One(T2), One (T4), Rest (T3);
R22	0.3333^{N}	All (T2)
	$+N\times0.6666\times0.3333^{(N-1)}$	Or One (T3,T4), Rest (T2)
R11	$[1-0.6666^{N}-0.6666^{N} -N\times0.3333\times0.6666^{(N-1)} - N\times0.3333\times0.6666^{(N-1)} +0.3333^{N} + N\times0.3333\times0.3333^{(N-1)} +N\times0.3333\times0.3333^{(N-1)} +N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}]$	Or Two(T4), Two(T2), Rest (all)
R12	$N\times0.3333\times[0.6666^{(N-1)}-0.3333^{(N-1)}-0.3333^{(N-1)}-(N-1)\times0.3333\times0.3333^{(N-2)}]$	Two(T4), One(T3), One(T2), Rest(T3,T4);
	$+ [0.6666^{N}-0.3333^{N}- 0.3333^{N} - N\times0.3333\times0.3333^{(N-1)} -N\times0.3333\times0.3333^{(N-1)}]$	Or Two(T4), Two(T3),Rest(T3,T4)
R21	$[0.6666^{N} -0.3333^{N}-0.3333^{N}- N\times0.3333\times0.3333^{(N-1)} -N\times0.3333\times0.3333^{(N-1)}]$	Two(T2), Two(T3), Rest(T2,T3);
	$+N\times0.3333\times[0.6666^{(N-1)} -0.3333^{(N-1)} - 0.3333^{(N-1)} -(N-1)\times0.3333\times0.3333^{(N-2)}]$	Or Two(T2), One(T3), One(T4), Rest(T2,T3);
R13	$N\times0.6666\times0.3333^{(N-1)}]$	One(T2,T3), Rest(T4)
R23	$N\times0.3333\times0.3333^{(N-1)}$	One(T2), Rest(T3)

5) Scenario 8

Based on Appendix 2 we know the BNE strategies of all three types and therefore all potential revenues. Figure 4.12 demonstrates all these potential revenues (the small triangles are potential revenues). Table 4.16(A) shows the amounts of all the potential revenues in Scenario 8, and Table 4.16(B) shows the likelihoods of all the potential revenues. The expected revenue is equal to the weighted sum of all potential revenues.

Figure 4.12 Revenues in Scenario 8

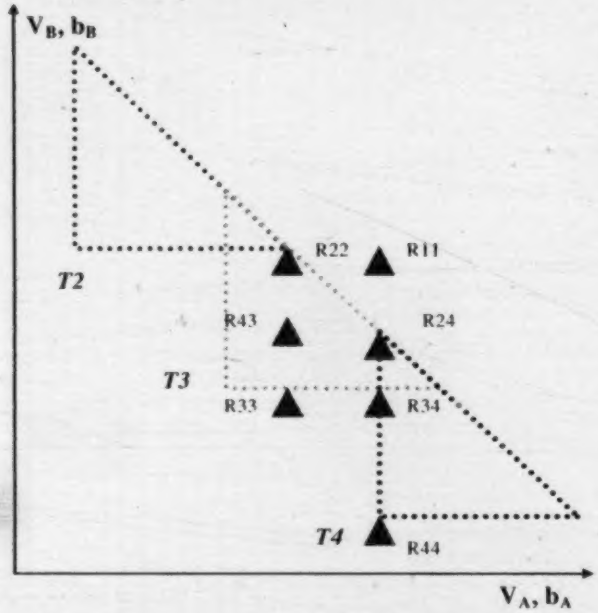


Table 4.16(A) Potential Revenues of Sequential Auctions in Scenario 8

Revenue	Amount
R44	120
R24	170
R33	130
R22	170
R11	200
R43	140
R34	160

Table 4.16(B) Potential Revenues of Sequential Auctions and Likelihoods in Scenario 8

Revenue	Likelihood	Explanation
R44	0.3333^N	All (T4);
R24	$N\times0.3333\times[0.6666^{(N-1)}-0.3333^{(N-1))}-(N-1)\times0.3333\times0.3333^{(N-2)}]$	One(T2), Two(T4), Rest(T3, T4)
R33	0.3333^N $+N\times0.3333\times0.3333^{(N-1)}$	All (T3); One (T4), Rest (T3);
R22	0.3333^N $+[0.6666^N-0.3333^N-0.3333^N]$ $+N\times0.3333\times[0.6666^{(N-1)}-0.3333^{(N-1))}-(N-1)\times0.3333\times0.3333^{(N-2)}]$	All (T2); Or One(T2),One(T3), Rest(T2,T3); Or One (T4), Two(T2),Rest (T2,T3);
R11	$[1-0.6666^N-0.6666^N$ $-N\times0.3333\times0.6666^{(N-1)}-N\times0.3333\times0.6666^{(N-1)}+0.3333^N+N\times0.3333\times0.3333^{(N-1)}+N\times0.3333\times0.3333^{(N-1)}+N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}]$	Or Two(T4), Two(T2), Rest (all)
R43	$N\times0.3333\times(N-1)\times0.3333\times0.3333^{(N-2)}$	One (T4),one(T2), Rest (T3);
R34	$[0.6666^N-0.3333^N-0.3333^N-N\times0.3333\times0.3333^{(N-1)}]$	Two(T4),One(T3), Rest(T3,T4)

Appendix 4.6: Instructions for Experiment

Welcome and thank you for participating in this research study. This study is about bidding behavior in auctions. You will be asked to place bids in a number of auctions and you will be able to earn cash by participating. Before we get started, please sign the research project consent form. You can keep one copy for your own records.

Second-price sealed-bid auction

In this study, you are going to bid in a series of **second-price sealed-bid auctions**. In a sealed-bid auction, each bidder places one bid per auction, and none of the bidders knows the bids of others when submitting his/her own bid.

The winner of the auction will be the bidder with the **highest bid**, and the price to be paid is the amount of the **second-highest bid**.

For example, consider an auction for Product A with the following five bids:

Bidder	**Michael**	Donna	Peter	Mary	Paul
Bid amount	$100	**$90**	$84	$80	$78

The outcome of this auction is as follows:

- Michael is the winner. (He submitted the highest bid.)
- Michael gets product A and he pays $90 (the amount of the second-highest bid).

How do you place bids and make money in this study?

All auctions in this study are for **hypothetical products**. This means that the winner of each auction receives an **amount of money** instead of an actual product. You will be told **in advance** what the **monetary value** of each of these hypothetical products is to you. If you win an auction, you will receive an amount equal to the **difference between your value for the product and the amount of the second-highest bid.**

In the example above, suppose that the **value** of Product A to the winner (Michael) happens to be **$100**. This means that

- Michael receives $10 (his value for Product A minus the amount of the second-highest bid).
- All other bidders receive $0.

To start out, you will be given a cash balance of $100 "**e-dollars**" in your "account". All your values and bids in the following auctions will be expressed in terms of e-dollars. All gains (losses) from auctions in this study will be added to (subtracted from) your account.

For every e-dollar in your account at the end of the study, you will actually receive **one cent in cash**. You will be notified by e-mail of the amount you are to receive, and of where you can pick up your payment. (The outcome of each auction will not be revealed during the experiment).

Quiz #1

*Note: For each correct answer, you gain **20 e-dollars** in your account.*

In **a single** second-price sealed-bid auction for the product A, there are 5 bidders, whose bids are as follows,

Bidder	John	Gerald	Peter	Paul	Jack
Bid amount	$50	$60	$65	$80	$25

1. The winner is _______________.

2. The applicable price for Product A is $_____.

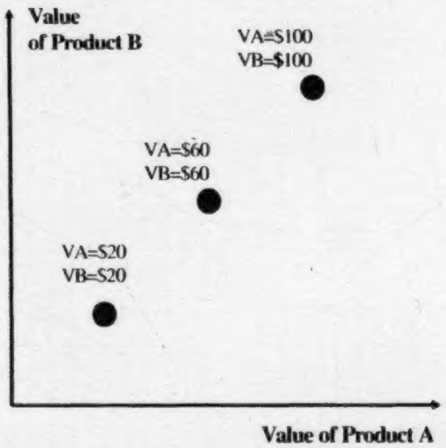
3. Suppose that the **value** of Product A to the winner happens to be **equal to his bid**. This means that, in this study, $_____ is added to the winner's account.

Now you will be participating in auctions for two hypothetical products A and B.

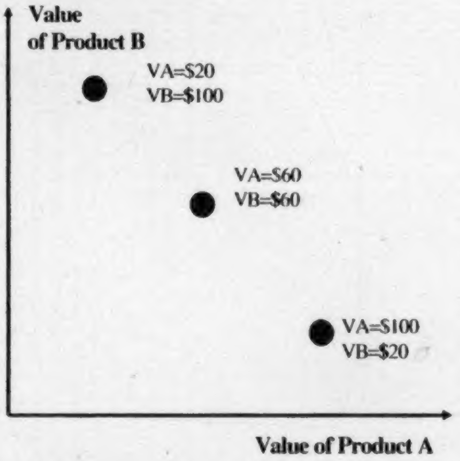
How are the values for these two products chosen?

In each auction, **your and your opponents' values** of Product A and Product B are randomly chosen from the **same distribution.** And in each auction, before bidding, each bidder will be told his/her values of Product A and Product B and the distribution from which his/her opponents' values are chosen. The following two distributions will be used in this study.

Distribution 1 (see figure below): Your values and your opponents' values for Product A and Product B will be randomly chosen from the following distribution with three pairs of values. So your values for Product A and Product B can be ($20, $20), ($60, $60), or ($100, $100) with **equal chance**. The same is true for your opponent's values for products A and B.



Distribution 2 (see figure below): Your values and your opponents' values for Product A and Product B will be randomly chosen from the following three pairs of values: ($20, $100), ($60, $60), and ($100, $20) with **equal chance**.



Bonus for winners of both products

In **each** auction, if any bidder wins **both** Product A and Product B, s/he will receive a **bonus**. The bonus may **vary** from auction to auction. In each auction, you will be told the amount of bonus before you bid, and the bonus is same to all bidders in this auction.

Who are your opponents?

In the following auctions, some of other participants are randomly chosen to be your opponents. The number of bidders may **vary** from auction to auction. In each auction, you will be told the number of bidders before you bid.

It is critical that there be no communication whatsoever during the course of this study!

One auction for the bundle consisting of Product A and Product B

In the next part of this study, we conduct second-price sealed-bid auctions for the **bundle** consisting of Product A and Product B.

You are required to submit only **one** bid for this bundle. The highest bidder is the winner.

If any bidder wins **both** Product A and Product B, s/he will receive a **bonus**.

Should there be a tie between the highest bids in any of these auctions, we will randomly pick one of the highest bidders to be the winner.

Now let's have a dry run for an auction for the bundle of products A and B.

Quiz (Bundle)

*Note: For each correct answer, you gain **20 e-dollars** in your cash balance.*

In **a single** second-price sealed-bid auction for **the bundle of Product A and Product B**, there are 3 bidders, whose bids are as follows,

Bidder	Michael	Peter	Gerald
Bid for bundle	$170	$140	$160

1. The winner is _______________.

2. The applicable price for the bundle is $_____.

Suppose the bonus of winning both A and B is $50.

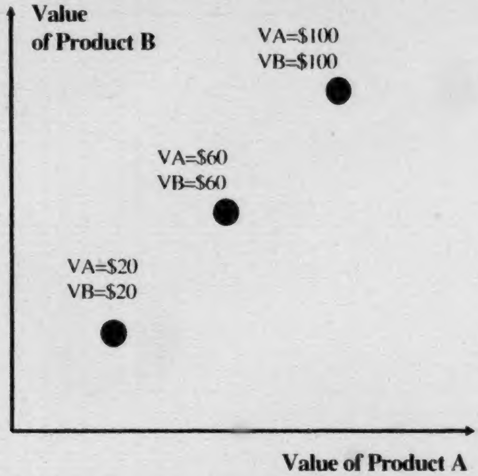
3. Suppose that the values of Product A and Product B to the winner are **$20** and **$100** respectively. This means that, in this study, $_____ is added to the winner's account.

4. Who gets the bonus? (Circle one)
    a) Gerald
    b) Peter
    c) Michael
    d) Nobody

Scenario 1: One second-price sealed-bid auction for the Bundle consisting of Products A and B

1. Number of bidders: 2

2. Your opponent's values for Product A and Product B are drawn **RAMDONLY** from the distribution in the figure below. In other words, your opponent's values have **EQUAL** chance to be one of the following three pairs.



3. The bonus for the winner of the bundle is: $20

For this scenario, there are 3 auctions. In each of these 3 auctions, your values of the two products are chosen to be one of the three pairs of values shown above. Please submit a bid for the bundle in all 3 auctions. Only **one** of the 3 auctions will be randomly chosen to be conducted. Please note that no matter what your values are, when you win both products you will receive the bonus.

	In this auction, my values for Product A and Product B are	I bid ... on the bundle
Auction 1	**1) $20, $20**	$____________
Auction 2	**2) $60, $60**	$____________
Auction 3	**3) $100, $100**	$____________

Two simultaneous separate auctions

In the next part of this study, we conduct **two simultaneous** second-price sealed-bid auctions for Product A and Product B.

A and B are sold in **two independent auctions**. You are required to submit a bid for each of two products. These two auctions are simultaneous in the sense that you must submit both bids before knowing the outcome of either auction.

If any bidder wins **both** Product A and Product B, s/he will receive a **bonus**.

Should there be a tie between the highest bids in any of these auctions, we will randomly pick one of the highest bidders to be the winner.

Now let's have a dry run of two simultaneous separate auctions.

Quiz (Simultaneous)

Note: For each correct answer, you gain ***20 e-dollars*** *in your cash balance.*

In **two simultaneous** second-price sealed-bid auctions for Product A and Product B, there are 3 bidders, whose bids are as follows.

Bidder	Gerald	Peter	Paul
Bid in auction for A	$70	$80	$100
Bid in auction for B	$100	$80	$70

1. The winner of product A is _______________

2. The winner of product B is _______________

Suppose the bonus of winning both A and B is $50.

3. Who gets the bonus? (circle one)

a) Gerald
b) Peter
c) Paul
[✓ d) Nobody]

If your answer to question 3 is (d), please answer questions 4 and 5.

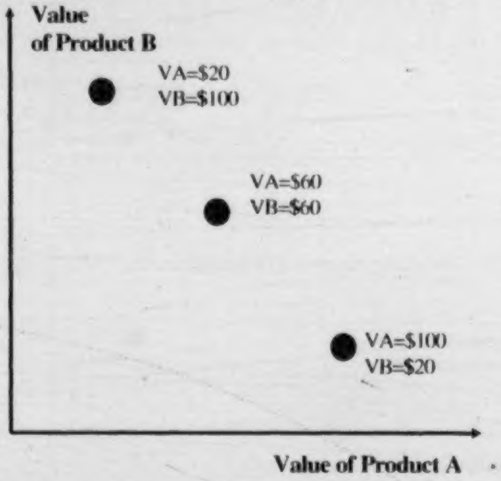
4. Suppose that the values of Product A and Product B to **the winner of Product A** are **$100** and **$20** respectively. This means that, in this study, $_____ is added to the account of **winner of Product A**.

5. Suppose that the values of Product A and Product B to **the winner of Product B** are **$20** and **$100** respectively. This means that, in this study, $_____ is added to the account of **winner of Product B**.

Scenario 1: two simultaneous second-price sealed-bid auctions

1. Number of bidders: 2

2. Your opponent's values for Product A and Product Bare drawn **RAMDONLY** by a computer from the distribution in the figure below. In other words, your opponent's values have **EQUAL** chance to be one of the following three pairs.



3. The bonus for the winner of both A and B is: $20

For this scenario, there are 3 auctions. In each of these 3 auctions, your values of the two products are chosen to be one of the three pairs of values shown above. Please submit a bid for the bundle in all 3 auctions. Only **one** of the 3 auctions will be randomly chosen to be conducted. Please note that no matter what your values are, when you win both products you will receive the bonus.

	In this auction, my values for Product A and Product B are	I bid ... on products A and B
Auction 1	**1) $100, $20**	$_______on A, $_______on B
Auction 2	**2) $60, $60**	$_______on A, $_______on B
Auction 3	**3) $20, $100**	$_______on A, $_______on B

Two sequential separate auctions

In the next part of this study, we conduct **two sequential second-price sealed-bid auctions**, with the **first** auction for Product **A** and the **second** auction for Product **B**.

A and B are sold in **two independent auctions**. You are required to place **a bid** for product **A** in **the first** auction and **two** bids for product **B** in the **second auction**: one bid if you were to **win** the first auction and one if you were to **lose** the first auction.

These two auctions are sequential in the sense that the outcome of the first auction for Product A decides which of your two bids for Product B will be used in the second auction for Product B.

If any bidder wins **both** Product A and Product B, s/he will receive a **bonus**.

Should there be a tie between the highest bids in any of these auctions, we will randomly pick one of the highest bidders to be the winner.

Now let's have a dry run of two sequential separate auctions.

Quiz (Sequential)

*Note: For each correct answer, you gain **20 e-dollars** in your cash balance.*

In **two sequential** second-price sealed-bid auctions for Product A and Product B (A is sold in the first auction), there are 3 bidders, whose bids are as follows,

Bidder	Gerald	Peter	Paul
Bid in 1st auction for A	$30	$70	$110
Bid in 2nd auction for B, *if A was **won***	$110	$70	$30
Bid in 2nd auction for B, *if A was **not** won*	$100	$60	$20

1. The winner of product A is ________________

2. The winner of product B is ________________

Suppose the bonus of winning both A and B is $50

3. Who gets the bonus? (circle one)

   a) Gerald
   b) Peter
   c) Paul
   d) Nobody

If your answer to question 3 is (d), please answer questions 4 and 5.

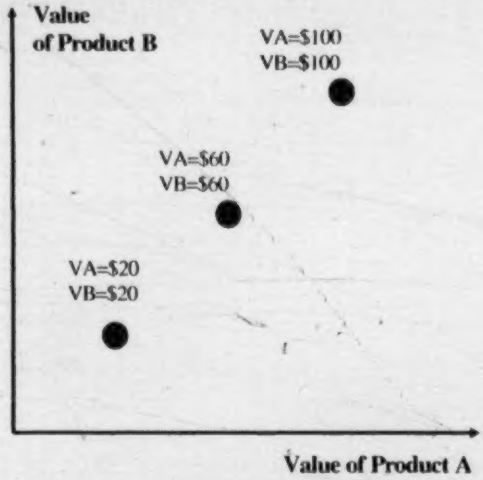
4. Suppose that the values of Product A and Product B to the **winner of Product A** are **$100** and **$20** respectively. This means that, in this study, $_____ is added to the account of **winner of Product A**.

5. Suppose that the values of Product A and Product B to the **winner of Product B** are **$20** and **$100** respectively. This means that, in this study, $_____ is added to the account of **winner of Product B**.

Scenario 1: two sequential second-price sealed-bid auctions

1. Number of bidders: 2

2. Your opponent's values for Product A and Product Bare drawn **RAMDONLY** by a computer from the distribution in the figure below. In other words, your opponent's values have **EQUAL** chance to be one of the following three pairs.



3. The bonus for the winner of both A and B is: $20

For this scenario, there are 3 auctions. In each of these 3 auctions, your values of the two products are chosen to be one of the three pairs of values shown above. Please submit a bid for the bundle in all 3 auctions. Only **one** of the 3 auctions will be randomly chosen to be conducted. Please note that no matter what your values are, when you win both products you will receive the bonus.

	In this auction, my values for Product A and Product B are	In the first auction I bid … on product A	In the second auction I bid … on product B
Auction 1	**1) $60, $60**	I bid $________on A.	If I win A. I bid $_______on B If I don't win A. I bid $_____on B
Auction 2	**2) $20, $20**	I bid $________on A.	If I win A. I bid $______on B If I don't win A. I bid $_____on B
Auction 3	**3) $100, $100**	I bid $________on A.	If I win A. I bid $______on B If I don't win A. I bid $_____on B

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Chapter 5

General Discussion

This thesis looks at two fundamental issues in bundling: how buyers evaluate product bundles and how sellers of multiple products sell these products. The answers to these questions are of tremendous theoretical and managerial significance. Overall, the work in this thesis makes the following significant contributions to marketing in both bundling and auction areas.

The first essay answers a fundamental question as to when a seller of multiple products should sell them in a bundled auction and when he should sell them in two simultaneous separate auctions. It is the first study in marketing and economics literature to incorporate all the three important factors - the degree of complementarity between the component products, the heterogeneity in bidders' product valuations, and the number of bidders - that affect bidders' bidding strategies and the relative profitability of a single auction of the bundle versus simultaneous separate auctions for the components. This essay bridges the gap between bundling literature and auction literature by showing that heterogeneity of values is not a one dimensional (measured by correlation coefficient) but a two dimensional variable with two independent components, variance and asymmetry. These two components of heterogeneity of individual values at group level affect people's bids and sellers' revenues. This essay investigates how exposure risk affects different bidders' strategies and sellers' revenues in separate auctions and provides the boundary conditions under which bundled auctions are more profitable.

Inspired by an unexpected empirical finding that people's value of a product bundle was lower than the value of a single product in the bundle, the second essay focuses on how buyers evaluate product bundles. It is the first study to examine how heterogeneity of certainty of values of individual products affects people's values of the bundle at an individual level. It clearly proves that decision makers evaluate individual products and then form the value of a bundle. More importantly, they make inferences about the value of low-certainty goods based on the value of high-certainty goods, leading to either hypersubadditivity or superadditvity in the values of product bundles even though the items are not complements or substitutes. The second essay contributes to bundling literature not only by reporting a counterintuitive but very important phenomenon, but also by exploring the mechanism underlying this phenomenon.

The third essay significantly extends the first essay by incorporating another important auctioning mechanism for multiple products, sequential separate auctions. It compares, through both analytical models and laboratory experiments, the profitability of three selling mechanisms in different environments that are characterized by the three factors in the first essay. It also investigates how the environment affects bidders' strategies for these three selling mechanisms. Specifically, it discusses how the environment affects bidders' perceived exposure risk and how the exposure risk affects bidders' overbidding in separate auctions. In a word, this essay is the first study in the auction literature to use an analytical model to investigate how strategic bidders bid under different conditions defined by the number of bidders, complementarity and heterogeneity of bidders' values for each of these three auction mechanisms.

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